Nonlinear Taxes for Spatially Mobile Workers

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Abstract:
This paper examines the effect of worker mobility on optimal nonlinear taxes. We devise a model in which two qualitatively different types of workers may choose to live in either of two jurisdictions. Residence decisions and labour-leisure choices are influenced by the tax schedules in effect in the two regions. Within worker category, individuals may differ in their attachment to one or the other of the regions. We show how the introduction of worker mobility affects the optimal tax schedule and address two related issues: the spatial distribution of the population under optimal taxation and how the optimal tax system depends on the existence inter-regional transfers.

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1. Introduction

Policy makers are conscious of the potential mobility of their workforce when deciding on tax policy. This is particularly so in the case of income taxation. It is sometimes argued that high rates of taxation can act as a catalyst for exit from a region. Much of the debate preceding the Canadian Federal budget of 2000, which included tax relief for high income earners, centered on potential migration of skilled workers to the United States. While it seems reasonable that the policy changes enacted in that budget might result in a larger skilled workforce in Canada, it is not clear *a priori* that it represents any improvement in the overall tax system. In this paper we attempt to model a nonlinear tax system in a way that allows a systematic analysis of this kind of policy measure.

We emphasise two characteristics of the economy that must exist for "brain drain" scenarios to be important. First, workers must have qualitative differences. Second, workers need to differ in their propensity to move between regions. If, for some reason, skilled workers are more likely to leave their region of origin than unskilled workers then special requirements may needed.

The model we present is a variant of the Stiglitz (1982) optimal nonlinear income tax model with endogenous wages. There are two types of labour used in production, and two regions in which production may take place. Within each region the relative wages of the two types of labour are determined, in part, by the supply of both types of labour. Labour supply depends on the labour choice of each worker and on the number of workers of a given type in a region. Following Mansoorian and Myers (1993), we allow workers to have a level of attachment to one or the other of the regions so that a region does not empty due to tiny differences in wage rates across borders. Instead, marginal changes in tax policy induce small changes in the composition of the workforces in each region. These changes affect both the size of the tax base and the relative wages in the two regions. The exact nature of these changes depends on how attachment to home varies across worker type and on the production technology.

Despite the relative complexity of the economic environment, some clear conclusions about the effect of residential mobility on tax policy are derived. We show that inter-regional mobility *per se* does not provide an additional motivation for marginal distortions. The choice among a finite set of places of residence is determined by a comparison of the total utility each place provides. Any desire a planner may have to change the composition of the population must be carried out by changing the distribution of total utility. Marginal distortions beyond those necessitated by imperfect information are an inefficient way to redistribute total utility. However, this does not mean that a change in residential mobility has no effect on the magnitude of marginal tax rates. As in Saez (2002a), the elasticity of the supply of each type of labour is affected by both changes in hours worked and changes in where work is carried out. Worker mobility has indirect effects through the labour supply.

While the qualitative features of tax schedules are not greatly affected by the mobility of workers, the spatial distribution of population is affected by tax schedules. Unless regions have identical compositions of workers, any attempt to redistribute between workers of differing skills is *de facto* a redistribution across borders. This, in turn, induces a movement of workers across borders. We show that this phenomenon can lead to identical (in terms of productivity) workers in differing regions receiving different allocations of goods at a utilitarian optimum. Moreover, it is sometimes impossible to prevent a redistribution between workers of differing skills within the same region from having spillover effects in another. Redistributions may change the relative incentives workers of differing skills have to move. A flow of workers in response to a redistribution in another region typically alters a region's own government budget balance. We argue that if compensating inter-regional budgetary transfers are not available then the optimal tax system may result in one of the jurisdictions operating inside its production possibilities frontier.

The remainder of the paper is organised as follows. The next section outlines the most general model considered in this paper. Sections 3 and 4 describe, respectively, the general properties of implementable and optimal tax systems. Section 5 focuses attention on an economy with fixed producer wages in which it is easier to present our results on the spatial distribution of workers and on the consequences of disallowing inter-regional budgetary transfers. Section 6 offers concluding remarks. Proofs are gathered in an Appendix.

2. The Model

The idealised economy considered in this paper consists of two regions, labelled A and B. These regions are populated by workers of two types, 1 and 2. We assume that the labels are assigned so that workers of type 2 earn higher wages. In general, each region is home to some portion of the individuals of each type. A single consumption good is produced is each region. For simplicity, we assume that it is the same good in each region. This good is called z. Production in region j is denoted z^j. Production requires the use of the labour of workers of both types, and is governed by the aggregate production functions

$$z^j = f^j(L_1^j, L_2^j), \quad j = A, B, \tag{1}$$

where L_i^j is the aggregate amount of labour of type i used in region j. An aggregate firm in each region is assumed to choose the mix of labour inputs in order to maximise profit. This maximisation process induces a demand for labour of each type. The aggregate supply of labour is influenced by the labour supply decisions of residents and by the (endogenous) number of residents of type i in region j. Wages are determined in this competitive environment.[1] The wage rate for workers of type i residing in region j is denoted w_i^j. The produced commodity is taken as the numeraire.

Workers are assumed to differ from one another in two ways. First, as is common in nonlinear income tax models, each has a skill type, either 1 or 2. Second, each worker has a residential preference parameter $\theta \in [0, 1]$. The lower the value of θ, the more the individual prefers to live in region A, all else equal. Individuals may have differing valuations of region-specific amenities, or they may possess a set of characteristics that allow them to achieve satisfaction more easily in one region than in the other. Language skills are an obvious example of the latter. Personal relationships or identification with a specific group

[1]Firms are assumed to maximise profit, so wage rates equal to values of marginal product of labour along the labour demand curves.

or community may also contribute the residential preference. An individual is identified by the ordered pair (i, θ) of his or her characteristics; this ordered pair is called his or her type. The total mass of individuals of each skill type is one. Within skill types, attachment to home is distributed according to the cumulative distribution functions $G_1(\theta)$ and $G_2(\theta)$. These distributions have density functions $g_1(\theta)$ and $g_2(\theta)$, respectively. The set of workers residing in region A is denoted $\mathcal{A}$; the set residing in region B, $\mathcal{B}$. $\mathcal{A}_i$ and $\mathcal{B}_i$ denote the set of workers of skill type i working in regions A and B, respectively.

Given their region of residence, workers choose labour supply. They have preferences over consumption of the produced good, x, labour supply, l, and region of residence represented by the function

$$U(x, l, \theta) = \begin{cases} v(x, l) + h(\theta), & \theta \in \mathcal{A} \\ v(x, l), & \theta \in \mathcal{B}. \end{cases} \tag{2}$$

The function $v(x, l)$ is assumed to be concave, increasing in x, decreasing in l and twice differentiable. It is a standard utility function. The function $h(\theta)$ measures additional utility accruing to an individual who chooses to reside in region A rather than region B, which is negative for those who prefer B to A. Consistency in the interpretation of the parameter θ requires that h be decreasing. We also assume that h is differentiable. Individuals are wage takers, so the before-tax income of a worker of skill type i residing in region j who supplies l units of labour is $y_i^j = w_i^j l$. Thus, preferences over x, y and θ are represented by

$$V(x, y, \theta) = \begin{cases} v\big(x, \frac{y}{w}\big) + h(\theta), & \theta \in \mathcal{A} \\ v\big(x, \frac{y}{w}\big), & \theta \in \mathcal{B}. \end{cases} \tag{3}$$

These preferences satisfy the standard single-crossing property in (x, y)-space.

A single taxation authority sets tax policy in the two regions simultaneously.[2] It cannot observe individual characteristics, nor can it observe labour supply. It can observe before-tax

[2]This rules out strategic behaviour on the part of governments in an attempt to produce a more desirable composition of population. While studying strategic behaviour is undoubtedly interesting, the case of coordinated tax policy is not yet understood. An understanding of the current problem should shed some light on what to expect in strategic settings.

income and choice of residence, but it cannot mandate hours or location of work. The best it can do is offer a schedule of before-tax and after-tax income combinations in each region. Individuals then choose their region of residence and hours of work to maximise utility. The resulting after-tax and before-tax income of person of type (i, θ) who chooses to reside in region j is denoted by $(x_i^j(\theta), y_i^j(\theta))$.[3] There is no *a priori* requirement that all individuals in a given region of the same work type choose the same allocation.

Because individual choices are made from an anonymous budget set, they satisfy the following self-selection conditions:

$$v\Big(x_i^A(\theta), \frac{y_i^A(\theta)}{w_i^A}\Big) + h(\theta) \geq v\Big(x_k^A(\hat{\theta}), \frac{y_k^A(\hat{\theta})}{w_i^A}\Big) + h(\theta), \quad \forall i, k = 1, 2,\ (\theta, \hat{\theta}) \in \mathcal{A}; \tag{4}$$

$$v\Big(x_i^A(\theta), \frac{y_i^A(\theta)}{w_i^A}\Big) + h(\theta) \geq v\Big(x_k^B(\hat{\theta}), \frac{y_k^B(\hat{\theta})}{w_i^B}\Big), \quad \forall i, k = 1, 2,\ \theta \in \mathcal{A},\ \hat{\theta} \in \mathcal{B}; \tag{5}$$

$$v\Big(x_i^B(\theta), \frac{y_i^B(\theta)}{w_i^B}\Big) \geq v\Big(x_k^B(\hat{\theta}), \frac{y_k^B(\hat{\theta})}{w_i^B}\Big), \quad \forall i, k = 1, 2,\ (\theta, \hat{\theta}) \in \mathcal{B}; \tag{6}$$

$$v\Big(x_i^B(\theta), \frac{y_i^B(\theta)}{w_i^B}\Big) \geq v\Big(x_k^A(\hat{\theta}), \frac{y_k^A(\hat{\theta})}{w_i^A}\Big) + h(\theta), \quad \forall i, k = 1, 2,\ \theta \in \mathcal{B}, \hat{\theta} \in \mathcal{A}. \tag{7}$$

Relations (4) and (6) state that any individual weakly prefers the allocation they receive to the allocation chosen by other persons residing in the same region. These are very similar to the incentive compatibility conditions found in one-region optimal taxation models. Relations (5) and (7) are less familiar. These are satisfied if an individual prefers his or her allocation to what could be received in the other region. The best alternative for a potential migrant need not be the allocation designed for members of its work type residing in another region. Incentive compatibility includes the requirement that a person not be tempted to simultaneously change region of residence and misrepresent his or her underlying skill. In the language of nonlinear tax models, individuals must not find it in their interest to move-and-mimick.

The total supply of labour of a given skill type in a particular region comprises the

[3] Lemmas 1-3 below make it clear that it is not necessary to specify $(x_i^j(\theta), y_i^j(\theta))$ if individual (i, θ) does not reside in region j. We make the very weak assumption that the functions $x_i^j(\theta)$ and $y_i^j(\theta)$ are integrable with respect to $G_i(\theta)$.

time spent at work of all workers of that skill type who choose to live in that region. In a competitive equilibrium, aggregate labour demand does not exceed labour supply, or

$$L_i^A \leq \int_{\theta\in\mathcal{A}_i} \frac{y_i^A(\theta)}{w_i^A} d\theta, \quad i = 1,2 \quad \text{and} \quad L_i^B \leq \int_{\theta\in\mathcal{B}_i} \frac{y_i^B(\theta)}{w_i^B} d\theta, \quad i = 1,2. \tag{8}$$

3. Incentive Compatible Taxation

The implications of anonymous taxation on the structure of allocations in a one-region economy are well-documented.[4] As we will soon show, many of the canonical results carry over to economies with mobile residents. However, the existing literature is silent on the nature of residential choices. We now turn our attention to examining the incentive compatible distributions of population.

While residential preferences are continuously distributed, the choice of locations is limited. The taxation authority has no instrument in its arsenal to target location preference directly. For instance, if θ reflects linguistic ability alone, the government does not give tax credits for language training or capabilities. Given the preferences (3), the marginal rate of substitution between x and y is independent of θ. Thus, labour-leisure choices do not vary systematically with θ. The possibility of differentiating among individuals of the same skill type settled in a particular region according to residential preference is severely limited, as the following Lemma makes explicit.

Lemma 1 The self-selection constraints imply that all individuals of the same skill type living in the same region receive the same utility from consumption and labour. Specifically,

$$v\Big(x_i^j(\theta), \frac{y_i^j(\theta)}{w_i^j}\Big) = v\Big(x_i^j(\hat{\theta}), \frac{y_i^j(\hat{\theta})}{w_i^j}\Big) \quad \forall i = 1,2,\ j = A,B,\ \theta,\hat{\theta} \in [0,1]. \tag{9}$$

Lemma 1 states that all individuals of a given skill type residing in the same region are on the same indifference curve in (x, y)-space. If (3) is interpreted as a numerical representation of utility then low-θ individuals living in the region A are better off than their high-θ

[4] See Guesnerie and Seade (1982), Stiglitz (1982) and Weymark (1986) for thorough treatments of this issue.

compatriots. Because it is impossible to discriminate on the basis of θ, there are rents to be gained from living in one's *a priori* more preferred location. The government has no means of measuring or taxing these rents. Lemma 1 leaves open the possibility of individuals of the same skill type choosing different allocations on the same (x, y)-indifference curve.

The location rents afforded to individuals residing in region A are decreasing in θ, all else equal. Lemma 1 guarantees that the non-location portion of utility is constant for all residents of region A of a given skill type. Thus, it seems intuitive — and the following Lemma confirms — that if an individual of type (i, θ) chooses to live in region A then a person of type $(i, \hat{\theta})$ with $\hat{\theta} < \theta$ would also reside in region A.

Lemma 2 The self-selection constraints imply residential sorting according to θ for each work type. Specifically, there exists some θ_i^* such that $\mathcal{A}_i = [0, \theta_i^*]$, $i = 1, 2$.

All residents of region A face the same set of consumption-before tax income possibilities should they move to region B. Individuals of the same work-type have identical preferences over these allocations. If labour market opportunities in region B do not attract individuals with a relatively weak attachment to region A then those same opportunities cannot attract workers with a relatively strong attachment to region A.[5]

Lemma 2 can also be viewed from a screening perspective. The incentive compatibility constraints require that individuals not find it in their interest to move-and-mimick. Because workers may use the move-and-mimick strategy, the planner cannot design harsh penalties for workers who choose the "wrong" region. Such penalties can be avoided by claiming the allocation on offer for a worker of some "correct" residential preference. Moving to region B is, in a sense, an outside option for residents of region A. The nature of residential mobility requires this outside option to be implementable. As Jullien (2000) shows, the structure of feasible allocations is often greatly simplified when the outside option is implementable. Lemma 2 describes the exact nature of this simplification for the economy under study.

[5]Lemma 2 allows for θ_i^* to be zero or one, so the question of the existence of a person with a strong enough attachment to region A to reside there has no bearing on the validity of the Lemma.

If both jurisidictions are populated with individuals of each skill type, then θ_1^* and θ_2^* are determined by the condition

$$v\Big(x_i^A(\theta_i^*), \frac{y_i^A(\theta_i^*)}{w_i^A}\Big) + h(\theta_i^*) = v\Big(x_i^B(\theta_i^*), \frac{y_i^B(\theta_i^*)}{w_i^B}\Big), \tag{10}$$

which states that the marginal individual of each skill type is indifferent between the two locations. Equation (10) is perhaps the most natural indifference condition, as it relates the utility gained by individuals of the same skill type in the two locations. However, in order for (10) to describe the residential sorting in this economy, it must be the case that the move-and-mimick strategy is inferior to simply moving and consuming the bundle intended for individuals of one's own skill type in the other region. The following Lemma states, loosely, that movers have no incentive to mimick as long as non-movers have no incentive to mimick, thereby justifying equation (10).

Lemma 3 If the within-region and within-skill type self-selection constraints are satisfied then all the self-selection constraints are satisfied. Specifically, if (4) and (6) hold for all i and j and (5) and (7) hold for $i = k$ then (5) and (7) hold for $i \neq k$.

4. Optimal Taxation

The taxation authority has a Paretian objective function, defined over the utilities of all individuals residing in both regions. It designs tax policy to maximise the value of this objective subject to the incentive compatibility conditions and economy-wide (inter-regional) market clearing conditions. Technical feasibility of the final allocation of resources requires that aggregate consumption be no greater than aggregate production. The production sector of the economy is described by the relations (1) and (8). Using Lemma 2, the overall materials balance constraint is given by

$$\begin{aligned}&\int_0^{\theta_1^*} x_1^A(\theta)g_1(\theta)d\theta + \int_0^{\theta_2^*} x_2^A(\theta)g_2(\theta)d\theta + \int_{\theta_1^*}^1 x_1^B(\theta)g_1(\theta)d\theta + \int_{\theta_2^*}^1 x_2^B(\theta)g_2(\theta)d\theta \leq \\ &f^A\Big(\int_0^{\theta_1^*} \frac{y_1^A(\theta)}{w_1^A}g_1(\theta)d\theta, \int_0^{\theta_2^*} \frac{y_2^A(\theta)}{w_2^A}g_2(\theta)d\theta\Big) + f^B\Big(\int_{\theta_1^*}^1 \frac{y_1^B(\theta)}{w_1^B}g_1(\theta)d\theta, \int_{\theta_2^*}^1 \frac{y_2^B(\theta)}{w_2^B}g_2(\theta)d\theta\Big)\end{aligned} \tag{11}$$

As is the tradition in optimal tax models, we work with a materials balance constraint rather than with a government budget constraint. This requires the assumption of complete taxation of pure profit. Moreover, use of the aggregate constraint (11) implies that transfers of resources across regional boundaries are permitted and are carried out to an optimal level.

As noted in the discussion of Lemma 1, self-selection alone is not enough to guarantee that all individuals of the same skill type residing in the same region receive the same allocation. However, the planner can do no better than to grant the same allocation to all individuals of the same skill type in the same region. The following Lemma provides a justification for this strategy.

Lemma 4 For any allocation of resources $\{(x_i^j(\theta), y_i^j(\theta))\}_{j=A,B;\, i=1,2}$ satisfying (11) and (4)-(8) there exists an alternative allocation of resources $\{(\tilde{x}_i^j(\theta), \tilde{y}_i^j(\theta))\}_{j=A,B;\, i=1,2}$ for which the following statements are all true.

i. All individuals are indifferent between the original and alternative allocations.

ii. No individual changes regions between the original and alternative allocations.

iii. The alternative allocation satisfies (11) and (4)-(8).

iv. There exists $\tilde{x}_i^j$ and $\tilde{y}_i^j$ such that for all i and j: $\tilde{x}_i^j(\theta) = \tilde{x}_i^j$ and $\tilde{y}_i^j(\theta) = \tilde{y}_i^j$.

Lemma 4 is an extension of Proposition 2 of Brito et.al. (1990, p.67) to a continuous population. In their terminology, all workers of the same skill type living in the same region form a self-selection cycle, as each is indifferent between their allocation and that of any other member of this group. Lemma 4 states that the planner can bunch these workers at the same allocation at a solution to its optimisation problem. The proof of Lemma 4 makes it clear that such bunching creates a surplus of consumption goods. At this point, though, we have not shown that the planner can always feasibly dispense of such a surplus to make some workers better off. Thus, we have not ruled out optimal allocations of other kinds. Despite this possibility, we choose to focus on these bunching solutions.[6]

[6] In Section 5 below, we investigate the circumstances in which the overall feasibility conditions are binding.

In view of Lemma 4, the optimal taxation problem can be viewed as a requirement to select four (x, y) pairs — one for each skill type of worker in each region — and four wage rates. Although not choice variables *per se*, wage rates are determined endogenously. Thus, they are under the implicit control of the taxation authority. We shall focus on the case of a central taxation authority with utilitarian objectives. In this circumstance, the planner wishes to maximise

$$\begin{aligned}\mathcal{W} :=& v\Big(x_1^A, \frac{y_1^A}{w_1^A}\Big)G_1(\theta_1^*) + \int_0^{\theta_1^*} h(\theta)g(\theta)d\theta + v\Big(x_1^B, \frac{y_1^B}{w_1^B}\Big)[1 - G_1(\theta_1^*)] \\ &+ v\Big(x_2^A, \frac{y_2^A}{w_2^A}\Big)G_2(\theta_2^*) + \int_0^{\theta_2^*} h(\theta)g(\theta)d\theta + v\Big(x_2^B, \frac{y_2^B}{w_2^B}\Big)[1 - G_2(\theta_2^*)].\end{aligned} \tag{12}$$

The constraints faced by the planner take on relatively simple forms, given Lemma 4 and the results of Section 3. The materials balance constraint is given by

$$\begin{aligned}& x_1^A G_1(\theta_1^*) + x_1^B[1 - G_1(\theta_1^*)] + x_2^A G_2(\theta_2^*) + x_2^B[1 - G_2(\theta_2^*)] \\ \le & f^A\Big(\frac{y_1^A}{w_1^A}G_1(\theta_1^*), \frac{y_2^A}{w_2^A}G_2(\theta_2^*)\Big) + f^B\Big(\frac{y_1^B}{w_1^B}[1 - G_1(\theta_1^*)], \frac{y_2^B}{w_2^B}[1 - G_2(\theta_2^*)]\Big).\end{aligned} \tag{13}$$

The labour market equilibrium conditions are

$$\frac{y_i^A}{w_i^A}G_i(\theta_i^*) \ge L_i^A(w_1^A, w_2^A) \quad \text{and} \quad \frac{y_i^B}{w_i^B}[1 - G_i(\theta_i^*)] \ge L_i^B(w_1^B, w_2^B), \quad i = 1, 2. \tag{14}$$

Given the utilitarian nature of the objective function, the solution will be "redistributive" in the sense of Guesnerie (1995, pp.222-224), so that we consider only the downward self-selection constraints

$$v\Big(x_2^j, \frac{y_2^j}{w_2^j}\Big) \ge v\Big(x_1^j, \frac{y_1^j}{w_2^j}\Big), \quad j = A, B. \tag{15}$$

In view of equation (10) and Lemmas 3 and 4, the allocation of population across jurisdictions is governed by[7]

$$\theta_i^* = \phi(x_i^A, y_i^A, w_i^A, x_i^B, y_i^B, w_i^B) := h_i^{-1}\Big(v\Big(x_i^B, \frac{y_i^B}{w_i^B}\Big) - v\Big(x_i^A, \frac{y_i^A}{w_i^A}\Big)\Big), \quad i = 1, 2. \tag{16}$$

In these circumstances, only the bunching solutions prove to be optimal, as they require the government to dispense all surplus.

[7]We assume throughout that $\theta_i^* \in (0, 1)$ for $i = 1, 2$. This assumption is satisfied if both types of labour are essential for production in each region and/or residential preferences are sufficiently salient in workers' decision making. If $\theta_i^* \in \{0, 1\}$, then marginal changes in tax policy would typically have no affect on residential decisions for workers of skill type i.

To summarise, the planner's problem may be formulated as

$$\max_{x_1^A,x_1^B,x_2^A,x_2^B,y_1^A,y_1^B,y_2^A,y_2^B,w_1^A,w_1^B,w_2^A,w_2^B} \mathcal{W} \quad \text{s.t.} \quad (13)\text{-}(16). \tag{17}$$

The qualitative features of the optimal nonlinear income tax schedule are, perhaps surprisingly, very similar to those that obtain in the one-region nonlinear tax problem. In order to state these properties and to compare them with their single region counterparts, it is necessary to introduce a few more pieces of notation. Let λ be the shadow value of the constraint (13), also known as the shadow value (in utility terms) of public funds; let μ^j be the multiplier associated with the self-selection constraint in jurisdiction j, and let ψ_i^j be the shadow price of labour of skill type i in region j. Finally, let $v_{x_1}^j$ denote the marginal utility of consumption of workers of skill type 1 residing in region j and let $\hat{v}_{x_1}^j$ denote the marginal utility of consumption a worker of type 2 residing in region j would enjoy whilst mimicking a worker of skill type 1.[8] With this notation in mind, we state the main features of the tax schedule in jurisdiction A. The tax schedule in jurisdiction B has similar properties.

Proposition 1 The following statements are true at a solution to the planner's problem (17).

i. Workers of skill-type 2 residing in region A receive a marginal wage subsidy. In particular,

$$-\frac{v_{l_2}^A}{v_{x_2}^A} = w_2^A + \frac{\psi_2^A}{\lambda} > w_2^A. \tag{18}$$

ii. Workers of skill-type 1 residing in region A have marginal rate of substitution between labour and consumption given by

$$-\frac{v_{l_1}^A}{v_{x_1}^A} = \frac{w_1^A \lambda G_1(\theta_1^*) - \mu^A \hat{v}_{l_1}^A \frac{w_1^A}{w_2^A} + \psi_1^A G_1(\theta_1^*)}{\lambda G_1(\theta_1^*) + \mu^A \hat{v}_{x_1}^A}. \tag{19}$$

The results reported in Proposition 1 are exact replicas of those reported by Stiglitz (1982) for a single jurisdiction. As in Stiglitz, more highly skilled workers receive a wage subsidy.

[8]Analogous expressions for individuals of type 2, or denoting the marginal utility of labour also appear in the sequel. To save space, the reader is asked to complete the analogy.

The slight differences in form between equations (18) and (19) and their counterparts in Stiglitz is due to the allowance we make for decreasing returns to scale. The motivation for the marginal subsidy at the top is exactly the motivation outlined by Stiglitz: to increase the labour supply of highly skilled workers, thereby enhancing redistribution by reducing the wage gap between workers of different skills. Ostensibly, mobility has no effect on the formulae for the optimal tax rates. This echoes the findings of Wilson (1992), who studies the effects of mobility into or out of a single region on the optimal tax schedule in that region. In that situation, the optimal tax scheme also exhibits marginal subsidies at the top.

It stands to reason that residential decisions have no direct bearing on marginal tax rates. Marginal changes in residential choices have no direct utility consequences, as marginal residents are indifferent between the two jurisdictions. Moreover, marginal distortions in policy instruments are incapable of producing welfare-enhancing changes in residential decisions. On the one hand, location is determined by differences in total utility, not by differences in the marginal utilities of various commodities. Residential decisions may be altered by redistributing total utility. Additional marginal distortions are counter-productive, as they inhibit the efficient distribution of utility. On the other hand, marginal changes in policy instruments have a marginal effect on the spatial distribution of workers (hence, on relative wages and the demand for consumption), because the set of workers indifferent between locations is of measure zero. Saez (2002b) has shown the importance of the measure of indifferent workers on the qualitative features of optimal taxation in the context of occupational choice, which is a just a different kind of discrete labour market choice. Residential mobility has indirect effects, however, operating through the shadow value of labour and the marginal value of public expenditure.

5. Fixed Producer Wages

One way to isolate the effects of residential mobility on the optimal tax structure is to work with the familiar fixed wage model. This model specialises the productions functions to

$$z^j = w_1^j l_1^j + w_2^j l_2^j \quad j = A, B, \tag{20}$$

which exhibit constant returns to scale and linear isoquants in (l_1, l_2)-space. With these assumptions, the profit maximising choice of labour inputs is undefined and the labour market clearing conditions (8) are redundant. The planner no longer has even implicit control over producer wages, and the planner's problem reduces to

$$\max_{x_1^A, x_1^B, x_2^A, x_2^B, y_1^A, y_1^B, y_2^A, y_2^B} \mathcal{W} \quad \text{s.t.} \quad (15),\ (16), \text{ and}$$
$$[y_1^A - x_1^A]G_1(\theta_1^*) + [y_1^B - x_1^B][1 - G_1(\theta_1^*)] + [y_2^A - x_2^A]G_2(\theta_2^*) + [y_2^B - x_2^B][1 - G_2(\theta_2^*)] \geq 0. \tag{21}$$

The results contained in Proposition 1 carry over with some straightforward modifications. For completeness, we record the analogous results in the following Proposition.

Proposition 2 The following statements are true at a solution to the planner's problem (21).

i. Workers of skill-type 2 residing in region A face a zero marginal tax rate on labour income. In particular,
$$-\frac{v_{l_2}^A}{v_{x_2}^A} = w_2^A. \tag{22}$$

ii. Workers of skill-type 1 residing in region A pay a positive marginal rate of income tax. In particular,
$$-\frac{v_{l_1}^A}{v_{x_1}^A} = w_1^A \left(\frac{\lambda G_1(\theta_1^*) - \mu^A \hat{v}_{l_1}^A \frac{1}{w_2^A}}{\lambda G_1(\theta_1^*) + \mu^A \hat{v}_{x_1}^A} \right) < w_1^A. \tag{23}$$

Proposition 2 recasts the main findings of Proposition 1. Even if the planner wishes to induce some migration by highly skilled workers as part of the optimal scheme, marginal distortions are not to be recommended. Marginal distortions are an inefficient way to redistribute utility. The familiar positive marginal tax rate at the bottom is easily seen in this version of the model. It arises for exactly the same reason as the distortion in a one-region model: asymmetric information about work type.

5.1. The effects of redistributive taxation on residential decisions

While differences in marginal rates of taxation are not sufficient to cause migration, differences in real living standards are. The optimal tax system may induce a distribution of population different from what would prevail under laissez-faire. While the model presented here is too general to provide a straightforward comparison of the no-tax population distribution with the second-best optimal distribution, some insight into the forces at play is gained by studying a further specialisation of the model that features identical wage distributions in the two regions and a particular normalisation of the residential attachment functions.

Assumption 1 $w_i^A = w_i^B$ and $h_i(\frac{1}{2}) = 0$, for $i = 1, 2$.

The laissez-faire outcome under Assumption 1 is easy to describe. Workers in each region choose identical work-consumption bundles and the mass of workers of types 1 and 2 residing in region A are $G_1(\frac{1}{2})$ and $G_2(\frac{1}{2})$, respectively. The within-region composition of the workforce is determined by the shapes of the distributions $G_1(\theta)$ and $G_2(\theta)$. Moreover, equality of wages implies it is always possible to select $(x_i^A, y_i^A) = (x_i^B, y_i^B)$ for $i = 1, 2$ in the second-best problem, ensuring a post-tax distribution of population identical to the laissez-faire distribution. The two self-selection constraints would then collapse to one. The next Proposition states, however, that it optimal to have identical allocations in the two regions only under very special circumstances.

Proposition 3 Let Assumption 1 hold. Then $(x_i^A, y_i^A) = (x_i^B, y_i^B)$ for $i = 1, 2$ holds at a solution to (21) only if

$$\frac{G_1(\frac{1}{2})}{1 - G_1(\frac{1}{2})} = \frac{G_2(\frac{1}{2})}{1 - G_2(\frac{1}{2})}. \tag{24}$$

Proposition 3 states that individuals of the same skill type living in different regions are bunched at the optimum only if the relative shares of workers of each type under laissez-faire is the same in each region. Differentiated allocations across regions induces an allocation of population different than under laissez-faire. The only way for workers of skill type 2 to simultaneously face a zero marginal income tax rate and be on the same indifference curve in (x, y)-space is to consume the same bundle. Hence, according to Proposition 3, they must be on different indifference curves in (x, y)-space, and $h(\theta_2^*)$ cannot be zero at the optimum.[9] The key to unlocking the intuition behind Proposition 3 is this close relationship between identical allocations and the laissez-faire distribution of population. The planner does not desire a redistribution of population for its own sake, nor does it necessarily wish to make a transfer from one region to another. However, when (24) is violated a redistribution of income between skill types causes a redistribution of income between regions. For example, if region A has a larger share of type 1 residents than region B does, a redistribution toward type 1 workers transfers purchasing power from region B to region A. This increased purchasing power in region A induces migration into region A. The planner is indifferent to marginal changes in the spatial distribution of population because the marginal individuals are indifferent between the two locations. Hence, it is willing to accept a small distortion in residential choice in order to enhance transfers to the less skilled.

Despite addressing very similar issues — namely, within-skill-type bunching — Lemma 4 and Proposition 3 generate very different results. Lemma 4 follows from the intuition of Brito et.al. (1990) that if two workers are at different points on the same indifference curve in (x, y)-space then the planner might just as well offer each of them the cheaper of the two bundles. The self-selection conditions require that all workers of the same skill type residing in the same region be on the same indifference curve. Because region of residence is observable, incentive compatibility does not require workers of the same skill type residing in different regions to be on the same indifference curve. Indeed, Proposition 3 implies that such workers are seldom on the same indifference curve at a second-best optimum.

5.2. Prohibiting inter-regional budgetary transfers

The analysis presented so far does not rest heavily on the interpretation of A an B as separate jurisdictions. Many of the results can be applied to occupational choice. Our

[9]While the statement of Proposition 3 does not rule out differentiation among workers of skill type 1 alone, such an outcome would either violate a self-selection constraint or leave one slack.

focus on an economy-wide budget constraint is largely responsible for this multiplicity of interpretation. While implicit (or even explicit) transfers of resources across workers in different occupations are to be expected in any redistributive tax system, transfers across jurisdictions are seldom observed. Even when such transfers are observed, it is difficult to assume that they are optimally set. A simple way to model sub-optimal transfers is to rule out cross-border monetary flows. A planner without the power to make inter-regional transfers no longer faces the budget constraint (13). Instead, under the assumption of fixed producer wages, the planner must satisfy the more stringent conditions

$$\begin{aligned} [y_1^A - x_1^A]G_1(\theta_1^*) + [y_2^A - x_2^A]G_2(\theta_2^*) &\geq 0, \text{ and} \\ [y_1^B - x_1^B][1 - G_1(\theta_1^*)] + [y_2^B - x_2^B][1 - G_2(\theta_2^*)] &\geq 0. \end{aligned} \tag{25}$$

The conditions (25) force materials balance on a region-by-region basis. The second-best utilitarian optimum of such an economy is a solution to the problem

$$\max_{x_1^A, x_1^B, x_2^A, x_2^B, y_1^A, y_1^B, y_2^A, y_2^B} \mathcal{W} \quad \text{s.t.} \quad (15),\ (16),\ \text{and}\ (25). \tag{26}$$

Cebreiro (2002) has investigated the effect of prohibiting inter-regional transfers in world-wide optimal tax settings using a slightly different model. In her model individuals do not differ in residential preference, so small changes in tax policy may induce a large migration across inter-regional borders. This possibility restricts the ability of a planner wishing to redistribute social surplus within a single region. An attempt to give this surplus to the more highly skilled in region A, say, may cause an undesirably large influx of highly skilled workers from region B. Because inter-regional transfers are not permitted, this surplus may not be partially diverted directly to region B in order to stem the tide. As is the case in one-region optimal tax models, an attempt to redistribute social surplus to the lesser-skilled individuals is not incentive compatible, in addition to creating the same potential for migration. Thus, she finds that one of the materials balance conditions (25) is typically slack at the optimum. Consequently, the overall materials balance condition is also slack, and the allocation of consumption is not on the production possibilities frontier. With diversity of residential preference, small changes in policy induce, at most, small changes in the spatial distribution of workers. Hence, as the next Proposition elucidates, it is possible to describe situations in which the economy operates on its production possibilities frontier at a second-best optimum.

Proposition 4 At least one of the constraints (25) binds at a solution to (26). Moreover, both parts of (25) bind at a solution to (26) if the following conditions hold:

$$\frac{v^A_{x_2}}{\hat{v}^A_{x_2}} \geq -\frac{[y^B_2 - x^B_2]g_2(\theta^*_2)\phi'_2 v^A_{x_2}}{[y^B_1 - x^B_1]g_1(\theta^*_1)\phi'_1 v^A_{x_1}} \quad \text{and} \tag{27}$$

$$\frac{v^B_{x_2}}{\hat{v}^B_{x_2}} \geq -\frac{[y^A_2 - x^A_2]g_2(\theta^*_2)\phi'_2 v^B_{x_2}}{[y^A_1 - x^A_1]g_1(\theta^*_1)\phi'_1 v^B_{x_1}}. \tag{28}$$

At first glance, conditions (27) and (28) are rather complicated. We concentrate on the interpretation of (27) to bring out their meaning. Suppose that at some candidate optimum, the materials balance constraint for region A does not bind. (Thus, by the first part of Proposition 4, the materials balance constraint must bind in region B.) The planner can raise the utility of the citizens of region A by increasing either x^A_1 or x^A_2 (or both) without violating materials balance in region A. But such increases must maintain the self-selection constraint and not violate materials balance in region B. When (27) holds, there exists a combination of increases in x^A_1 and x^A_2 that satisfy these conditions. The numerator on the right-hand side of (27) represents the change in resources used in region B due to a one-unit increase in x^A_2. This change is brought about by migration of higher skill-type individuals into region A. This revenue effect is easily interpreted. A one-unit change in x^A_2 increases v^A_2 by $v^A_{x_2}$, causing a $\phi' v^A_{x_2}$ change in the difference in utility between the two regions. Multiplying by the density of marginal individuals, $g_2(\theta^A_2)$, gives the total outflow of highly skilled workers from region B. Each of these workers contributes $[y^B_2 - x^B_2]$, the excess of pre-tax to after-tax income, to the public coffers in region B. If there is redistribution from workers of skill type 2 to workers of skill type 1 in region B, then this effect is negative. The denominator in (27) is the analogous, but typically positive, effect on the budget in region B of increases in x^A_1. Such increases attract to region A workers who would otherwise receive a net subsidy in region B. To ease notation, let Δ^B_2 and Δ^B_1 denote, respectively, the numerator and denominator on the right-hand side of (27). Then the planner can feasibly redistribute excess goods in region A if there exist a positive (dx_1^A, dx_2^A) such that

$$v_{x_2}^A dx_2^A - \hat{v}_{x_1}^A dx_1^A \geq 0, \quad \text{and} \quad \Delta_2^A dx_2^A + \Delta_1^A dx_1^A \geq 0. \tag{29}$$

The two inequalities in (29) describe the directions of change that maintain the self-selection and budget constraints, respectively. The condition (27) implies that (29) has a positive solution.

Figure 1 shows a configuration in which it is possible to satisfy both conditions of (29) simultaneously. The directions of change that maintain incentive compatibility are represented by the (dx_1^A, dx_1^B) combinations above the line perpendicular to the vector $(-\hat{v}_{x_1}^A, v_{x_2}^A)$, SS; materials balance in region B is maintained when (dx_1^A, dx_1^B) lies below the line BB, which is perpendicular to the vector (Δ_1^B, Δ_2^B). Because the slope of BB is greater than the slope of SS, incentive compatibility and budget balance can be simultaneously maintained by some (dx_1^A, dx_1^B) in the positive quadrant. This condition on the relative slopes is captured algebraically by (27). To imagine a situation in which (27) is violated, rotate BB clockwise (say, by moving Δ_2^B further away from the origin). It will eventually fall below SS in the positive quadrant, implying that all (dx_1^A, dx_1^B) satisfying (29) lie in the negative quadrant. But such directions neither increase welfare nor diminish the surplus on hand in region A.

FIGURE 1 ABOUT HERE

While Proposition 4 provides an interpretable sufficient condition for the economy to be operating on the production possibilities frontier at a second-best optimum, conditions (27) and (28) are littered with endogenous variables. Some simplification is afforded when the utility function $v(x, y)$ is additively separable. In that case, the mimicker and individuals of skill type 1 have the same marginal utility of income at any allocation, and the terms in the marginal utility of income cancel from (27) and (28). Moreover, a binding budget constraint in region B implies

$$-\frac{[y_2^B - x_2^B]}{[y_1^B - x_1^B]} = \frac{1 - G_1(\theta_1^*)}{1 - G_2(\theta_2^*)} \tag{30}$$

Then (27) and (28) reduce to

$$\frac{g_1(\theta_1^*)}{1 - G_1(\theta_1^*)}\phi_1' \geq \frac{g_2(\theta_2^*)}{1 - G_(\theta_2^*)}\phi_2' \quad \text{and} \quad \frac{g_1(\theta_1^*)}{G_1(\theta_1^*)}\phi_1' \geq \frac{g_2(\theta_2^*)}{G_2(\theta_2^*)}\phi_2'. \tag{31}$$

In general, the optimal allocation determines both θ_i^* and the argument of $\phi_i(\cdot)$, but there are certain special cases in which the endogeneity of these quantities in no way affects the interpretation of (31). Suppose, for example, that the residential preference functions $h_i(\cdot)$ (and, hence, $\phi_i(\cdot)$) are linear and that both $G_1(\theta)$ and $G_2(\theta)$ have constant hazard rates. Then (31) states that the economy optimally operates on its production possibilities frontier if the product of the hazard rate with the residential response to utility changes for workers of skill type 1 is larger than the analogous response for workers of skill type 2, and a similar condition also holds for the ratio of the densities to the distribution functions. Moreover, these two conditions are consistent. A necessary condition for (31) to hold is[10]

$$g_1(\theta_1^*)\phi_1' \geq g_2(\theta_2^*)\phi_2'. \tag{32}$$

Condition (32) is most easily interpreted when θ is uniformly distributed for both skill types and residential preferences are linear. It then says that both parts of (25) can bind at an optimum if workers of skill type 1 are more spatially responsive to changes in utility than their type 2 counterparts; that is, if the more lowly skilled are less attached to home than the more highly skilled. Loosely speaking, this gives the planner more freedom to dispose of surplus in an incentive compatible way by offering it to workers of type 2. If, on the other hand, highly skilled workers are also the more footloose, then one of the regions may be operating inside its production possibilities frontier.

6. Conclusion

This paper has presented a model of optimal taxation when workers of all types are potential migrants. Our focus has been on describing the economy-wide optimal tax system.

[10]To obtain this condition, cross multiply the two conditions in (31) and add.

We find that potential migration does not overturn the main findings of the optimal tax literature regarding the qualitative features of the marginal tax rates. The choice of where to work and live is not a marginal decision, so changes in the marginal tax rate *per se* have no effect on location decisions. Redistribution of total income across borders, however, does encourage migration. We have shown how redistribution across income classes often necessitates redistribution across borders. This, in turn, implies that the laissez-faire distribution of population may not be a feature of an economy with the optimal tax mix. We have also offered some sufficient conditions for the economy as a whole to operate on its production possibilities frontier in the absence of inter-regional transfers. These conditions require that the migration of highly skilled workers not exert too great a fiscal drain on the source region.

Because this is a work in normative economics, we view our results as providing a possible benchmark against which to assess the outcomes of strategic models of taxation, rather than as a description of any existing policy situation. Yet, our analysis has uncovered some economic forces that can shape policy. In Canada, for example, the Federal government is largely (though not entirely) responsible for personal income taxation. It also controls redistributive levers such as Employment Insurance. Because workforce composition differs among the provinces and territories, redistributions across income classes are implicit inter-regional transfers. There is nothing in our results to suggest that the consequences this may have for the distribution of workers across provinces are necessarily efficiency destroying. On the other hand, the difficulty of designing a federal system that respects provincia budget constraints is highlighted by our results. While our model considers the case of separate income-tax accounts for each province, it is easy to imagine that migration of highly skilled workers might affect any tax base occupied by provincial governments. If a federal government were forced to take the consequences of its decisions on provincial coffers into account the features of the tax system may be significantly altered.

Appendix

Proof of Lemma 1:

Take any $i = 1, 2$, $j = A, B$ and $\theta, \hat{\theta} \in [0, 1]$. By (4),

$$v\Big(x_i^j(\theta), \frac{y_i^j(\theta)}{w_i^j}\Big) - v\Big(x_i^j(\hat{\theta}), \frac{y_i^j(\hat{\theta})}{w_i^j}\Big) \geq 0; \tag{33}$$

$$v\Big(x_i^j(\hat{\theta}), \frac{y_i^j(\hat{\theta})}{w_i^j}\Big) - v\Big(x_i^j(\theta), \frac{y_i^j(\theta)}{w_i^j}\Big) \geq 0 \tag{34}$$

The left-hand side of (33) is the additive inverse of the left-hand side of (34), so both of these expressions must be zero. Equation (9) follows for workers in region A. A similar argument using (6) establishes the result for workers residing in region B. $\square$

Proof of Lemma 2:

The Lemma is false only if there exists a pair $((i, \theta), (i, \hat{\theta}))$ with $\hat{\theta} > \theta$, $(i, \theta) \in \mathcal{B}_i$ and $(i, \hat{\theta}) \in \mathcal{A}_i$ Suppose, by way of contradiction, that such a pair exists. By (5),

$$v\Big(x_i^A(\hat{\theta}), \frac{y_i^A(\hat{\theta})}{w_i^A}\Big) + h(\hat{\theta}) \geq v\Big(x_i^B(\theta), \frac{y_i^B(\theta)}{w_i^B}\Big). \tag{35}$$

By (7),

$$v\Big(x_i^B(\theta), \frac{y_i^B(\theta)}{w_i^B}\Big) \geq v\Big(x_i^A(\hat{\theta}), \frac{y_i^A(\hat{\theta})}{w_i^A}\Big) + h(\theta). \tag{36}$$

Combining (35) and (36) gives

$$v\Big(x_i^A(\hat{\theta}), \frac{y_i^A(\hat{\theta})}{w_i^A}\Big) + h(\hat{\theta}) \geq v\Big(x_i^A(\hat{\theta}), \frac{y_i^A(\hat{\theta})}{w_i^A}\Big) + h(\theta), \tag{37}$$

which implies $h(\hat{\theta}) \geq h(\theta)$. This contradicts the decreasingness of h. $\square$

Proof of Lemma 3:

Only the proof of that (5) holds for $i \neq k$ is presented. The proof regarding (7) is identical, save for notation.

Select $\theta \in \mathcal{A}$, $\hat{\theta} \in \mathcal{B}$, and $i \neq k$. By hypothesis, (5) holds for any two workers of the same skill type, so

$$v\Big(x_i^A(\theta), \frac{y_i^A(\theta)}{w_i^A}\Big) + h(\theta) \geq v\Big(x_i^B(\hat{\theta}), \frac{y_i^B(\hat{\theta})}{w_i^B}\Big). \tag{38}$$

By (6)

$$v\Big(x_i^B(\hat{\theta}), \frac{y_i^B(\hat{\theta})}{w_i^B}\Big) \geq v\Big(x_k^B(\hat{\theta}), \frac{y_k^B(\hat{\theta})}{w_i^B}\Big). \tag{39}$$

Combining inequalities (38) and (39) establishes the result. □

Proof of Lemma 4:

Consider the case of workers of skill type 1 residing in region A. Let $(x_1^A(\theta), y_1^A(\theta))$ be the part of a candidate optimal allocation designed for that group. This candidate optimum must satisfy the feasibility and self-selection constraints.

Hold the allocation for all other skill-residence combinations constant. By Lemma 1, there exits some constant k such that

$$v\Big(x_1^A(\theta), \frac{y_1^A(\theta)}{w_1^A}\Big) = k, \quad \forall \theta \in \mathcal{A}_1. \tag{40}$$

Define the function $x^k(y)$ by

$$x^k(y) = x \quad \leftrightarrow \quad v\Big(x, \frac{y}{w_1^A}\Big) = k \tag{41}$$

The graph of $x^k(\cdot)$ in (x, y)-space is the level set of the function $v(\cdot)$ on which all members of $(x_1^A(\theta), y_1^A(\theta))$ are found. The properties of $v(\cdot)$ ensure that $x^k(\cdot)$ is well-defined and strictly convex.

Because the allocation functions are bounded, there exists greatest lower bound and a least upper bound for for $y_1^A(\theta)$. Call these values $y^{\inf}$ and $y^{\sup}$, respectively. Because $\mathcal{A}_1$ is an interval, it is measurable. Its measure with respect to $dG(\theta)$ is $G(\theta^*)$. Thus, there exists a $\tilde{y}_1^A \in [y^{\inf}, y^{\sup}]$ such that[11]

$$G(\theta_1^*)\tilde{y}_1^A = \int_0^{\theta_1^*} y_1^A(\theta) dG(\theta). \tag{42}$$

If each worker in $\mathcal{A}_1$ were offered $\tilde{y}_1^A$ the total before-tax income (production) of this group would be the same as in the initial allocation.

Define $(\tilde{x}_1^A, \tilde{y}_1^A) = (x^k(\tilde{y}_1^A), \tilde{y}_1^A)$, which is the x-coordinate of the indifference curve on which workers in $\mathcal{A}_1$ are situated corresponding to the y-coordinate $\tilde{y}_1^A$. Then

$$\int_0^{\theta_1^*} \tilde{x}_1^A dG(\theta) = G(\theta_1^*) x^k(\tilde{y}_1^A) = G(\theta_1^*) x^k\Big(\int_0^{\theta_1^*} \frac{y_1^A(\theta)}{G(\theta_1^*)} dG(\theta)\Big). \tag{43}$$

[11] This result follows from Exercise 2 on p.86 of Berberian (1965).

Because $x^k(\cdot)$ is strictly convex, Jensen's inequality may be applied to (43) to conclude

$$\int_0^{\theta_1^*} \tilde{x}_1^A dG(\theta) < G(\theta_1^*) \int_0^{\theta_1^*} \frac{x^k(y_1^A(\theta))}{G(\theta_1^*)} dG(\theta) = \int_0^{\theta_1^*} x^k(y_1^A(\theta)) dG(\theta) \tag{44}$$

Thus replacing the candidate allocations for workers of skill type 1 residing in region A by $(\tilde{x}_1^A, \tilde{y}_1^A)$ results in an allocation with the same total before-tax income and lower aggregate consumption than the candidate allocation. Thus, the replacement allocation satisfies the materials balance constraint.

By construction, all members of $\mathcal{A}_1$ are indifferent between $(\tilde{x}_1^A, \tilde{y}_1^A)$ and their allocation in the candidate optimum. Thus, they (weakly) prefer $(\tilde{x}_1^A, \tilde{y}_1^A)$ to any allocation chosen by other individuals in the candidate optimum.

It remains to show that workers outside of $\mathcal{A}_1$ prefer their original allocations to $(\tilde{x}_1^A, \tilde{y}_1^A)$. This immediate if $(\tilde{x}_1^A, \tilde{y}_1^A)$ coincides with one of the original allocations for members of $\mathcal{A}_1$. Otherwise, select two elements of the original allocation $(x_1^A(\theta), y_1^A(\theta))$ and $(x_1^A(\theta'), y_1^A(\theta'))$ satisfying $(x_1^A(\theta), y_1^A(\theta)) \ll (\tilde{x}_1^A, \tilde{y}_1^A) \ll (x_1^A(\theta'), y_1^A(\theta'))$. Such a pair is guaranteed to exist unless all $(x_1^A(\theta), y_1^A(\theta))$ were equal in the original allocation. But single crossing of indifference curves in (x, y)-space ensures that, because all workers outside $\mathcal{A}_1$ prefer their original allocations to both $(x_1^A(\theta), y_1^A(\theta))$ and $(x_1^A(\theta'), y_1^A(\theta'))$, they must also prefer their original allocation to $(\tilde{x}_1^A, \tilde{y}_1^A)$. □

Proof of Proposition 1:

The proof relies heavily on the first order optimality conditions. We begin by presenting an argument that allows some simplification of the optimality conditions. The first order condition associated with x_1^A is

$$\begin{aligned} & v_{x_1}^A G_1(\theta_1^*) + v\Big(x_i^A, \frac{y_i^A}{w_i^A}\Big) g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} + h(\theta_1^*) g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} - v\Big(x_i^B, \frac{y_i^B}{w_i^B}\Big) g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} - \mu^A \hat{V}_{x_1}^A \\ & + \lambda\left[-G_1(\theta_1^*) - x_1^A g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} + x_1^B g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} + \frac{\partial f^A}{\partial l_1^A} \frac{y_1^A}{w_1^A} g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} - \frac{\partial f^B}{\partial l_1^B} \frac{y_1^B}{w_1^B} g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A}\right] \\ & + \left[\psi_1^A \frac{y_1^A}{w_1^A} - \psi_1^B \frac{y_1^B}{w_1^B}\right] g_1^*(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} = 0. \end{aligned} \tag{45}$$

By (10), the second through fourth terms of (45) sum to zero. Equality of the wage rate with marginal product in each region allows one to simplify the expression in square brackets, yielding

$$\begin{aligned} v_{x_1}^A G_1(\theta_1^*) - \lambda G_1(\theta_1^*) + \lambda\Big[(y_1^A - x_1^A - y_1^B + x_1^B) g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A}\Big] \\ + \left[\psi_1^A \frac{y_1^A}{w_1^A} - \psi_1^B \frac{y_1^B}{w_1^B}\right] g_1^*(\theta_1^*) \frac{\partial \theta_1^*}{\partial x_1^A} - \mu^A \hat{v}_{x_1}^A = 0. \end{aligned} \tag{46}$$

The same steps may be applied to any of the other optimality conditions. The first order conditions associated with x_2^A, y_1^A and y_2^A imply, respectively,

$$\begin{aligned} v_{x_2}^A G_2(\theta_2^*) - \lambda G_2(\theta_2^*) + \lambda\Big[(y_2^A - x_2^A - y_2^B + x_2^B) g_2(\theta_2^*) \frac{\partial \theta_2^*}{\partial x_2^A}\Big] \\ + \left[\psi_2^A \frac{y_2^A}{w_2^A} - \psi_2^B \frac{y_2^B}{w_2^B}\right] g_2^*(\theta_2^*) \frac{\partial \theta_2^*}{\partial x_2^A} - \mu^A v_{x_2}^A = 0; \end{aligned} \tag{47}$$

$$\begin{aligned} \frac{v_{l_1}^A}{w_1^A} G_1(\theta_1^*) + \lambda G_1(\theta_1^*) + \lambda\Big[(y_1^A - x_1^A - y_1^B + x_1^B) g_1(\theta_1^*) \frac{\partial \theta_1^*}{\partial y_1^A}\Big] + \frac{\psi_1^A}{w_1^A} G_1(\theta_1^*) \\ + \left[\psi_1^A \frac{y_1^A}{w_1^A} - \psi_1^B \frac{y_1^B}{w_1^B}\right] g_1^*(\theta_1^*) \frac{\partial \theta_1^*}{\partial y_1^A} - \frac{\mu^A}{w_2^A} \hat{v}_{l_1}^A = 0; \end{aligned} \tag{48}$$

$$\begin{aligned} \frac{v_{l_2}^A}{w_2^A} G_2(\theta_2^*) + \lambda G_2(\theta_2^*) + \lambda\Big[(y_2^A - x_2^A - y_2^B + x_2^B) g_2(\theta_2^*) \frac{\partial \theta_2^*}{\partial y_2^A}\Big] + \frac{\psi_2^A}{w_2^A} G_1(\theta_1^*) \\ + \left[\psi_2^A \frac{y_2^A}{w_2^A} - \psi_2^B \frac{y_2^B}{w_2^B}\right] g_2^*(\theta_2^*) \frac{\partial \theta_2^*}{\partial y_2^A} + \frac{\mu^A}{w_2^A} v_{l_2}^A = 0. \end{aligned} \tag{49}$$

By (16),

$$\frac{\partial \theta_i^*}{\partial x_i^A} = -h_i^{-1\prime}(\cdot) v_{x_i}^A \quad \text{and} \quad \frac{\partial \theta_i^*}{\partial y_i^A} = -h_i^{-1\prime}(\cdot) \frac{v_{l_i}^A}{w_i^A}, \quad i = 1, 2. \tag{50}$$

Thus, (47) and (49) imply

$$\begin{aligned} v_{l_2}^A \left[G_2(\theta_2^*) + \mu^A - \Big[\lambda(y_2^A - x_2^A - y_2^B + x_2^B) + \psi_2^A \frac{y_2^A}{w_2^A} - \psi_2^B \frac{y_2^B}{w_2^B}\Big] g_2(\theta_2^*) h_i^{-1\prime}(\cdot) \right] \\ + \psi_2^A G_2(\theta_2^*) = -\lambda w_2^A G_2(\theta_2^*); \end{aligned} \tag{51}$$

$$\begin{aligned} v_{x_2}^A \left[G_2(\theta_2^*) + \mu^A - \Big[\lambda(y_2^A - x_2^A - y_2^B + x_2^B) + \psi_2^A \frac{y_2^A}{w_2^A} - \psi_2^B \frac{y_2^B}{w_2^B}\Big] g_2(\theta_2^*) h_i^{-1\prime}(\cdot) \right] \\ = \lambda G_2(\theta_2^*). \end{aligned} \tag{52}$$

Dividing (51) by (52) gives equality in (18). Positiveness of ψ_2^A establishes the inequality in (18) A similar calculation using (46) and (48) establishes (19). $\square$

Proof of Proposition 2:

The equalities are easily derived from (18) and (19) by setting $\psi_j^i = 0$ for all i and j, which follow from the absence of the constraints (8). The proof of the inequality in (23) starts with recognising that the mimicker is on the same indifference curve as the agent of skill type 2, but that $(x_1^A, y_1^A) \ll (x_2^A, y_2^A)$, so that $\frac{-\hat{v}_{l_1}^A}{\hat{v}_{x_1}^A} < w_2^A$. Thus,

$$\lambda G_1(\theta_1^*) - \mu^A \hat{v}_{l_1}^A \frac{1}{w_2^A} < \lambda G_1(\theta_1^*) + \mu^A \hat{v}_{x_1}^A. \tag{53}$$

Hence, the term in parentheses in (23) is less than one, establishing the inequality. $\square$

Proof of Proposition 3:

Suppose that $(x_i^A, y_i^A) = (x_i^B, y_i^B)$ for $i = 1, 2$ at a solution to (2). Then, by Assumption 1, $\theta_1^* = \theta_2^* = \frac{1}{2}$. Moreover, the terms in square brackets in (46) and (47) are zero, yielding

$$v_{x_1}^A G_1\big(\frac{1}{2}\big) - \lambda G_1\big(\frac{1}{2}\big) = \mu^A \hat{v}_{x_1}^A, \tag{54}$$

$$v_{x_2}^A G_2\big(\frac{1}{2}\big) - \lambda G_2\big(\frac{1}{2}\big) = -\mu^A v_{x_2}^A, . \tag{55}$$

Likewise, the optimality conditions associated with x_1^B and x_2^B reduce to:

$$v_{x_1}^B \Big[1 - G_1\big(\frac{1}{2}\big)\Big] - \lambda \Big[1 - G_1\big(\frac{1}{2}\big)\Big] = \mu^B \hat{v}_{x_1}^B, \tag{56}$$

$$v_{x_2}^B \Big[1 - G_2\big(\frac{1}{2}\big)\Big] - \lambda \Big[1 - G_2\big(\frac{1}{2}\big)\Big] = -\mu^B v_{x_2}^B, . \tag{57}$$

Because the allocations are assumed to be symmetric across regions, we may divide (54) by (56) and (55) by (57) to give

$$\frac{G_1\big(\frac{1}{2}\big)}{1 - G_1\big(\frac{1}{2}\big)} = \frac{\mu^A}{\mu^B} = \frac{G_2\big(\frac{1}{2}\big)}{1 - G_2\big(\frac{1}{2}\big)}, \tag{58}$$

which establishes the result. $\square$

Proof of Proposition 4:

The first order conditions for (26) are identical to those for (21), except that the multiplier λ is replaced by a pair of multipliers (λ^A, λ^B). The former appears in first order conditions associated with goods in region A; the latter, in first order conditions associated with goods in region B. As in the text, we introduce the notation

$$\Delta_1^B := [y_1^B - x_1^B] g_1(\theta_1^*) \phi_1' v_{x_1}^A, \quad \Delta_2^B := [y_2^B - x_2^B] g_2(\theta_2^*) \phi_2' v_{x_2}^A. \tag{59}$$

First, we show that at least one of λ^A and λ^B is non-zero. Otherwise, the first order condition associated with x_2^A can be written

$$v_{x_2}^A \left[G_2(\theta_2^*) + \mu^A \right] = 0. \tag{60}$$

Because both terms on the left-hand side of (60) are positive, there is a contradiction. Thus, at least one of λ^A and λ^B is non-zero.

Now suppose that $\lambda^A = 0$ and $\lambda^B > 0$. The first order conditions associated with x_2^A and x_1^A become

$$v_{x_2}^A \left[G_2(\theta_2^*) + \mu^A \right] = -\lambda^B \Delta_2^B, \quad \text{and} \tag{61}$$

$$v_{x_1}^A G_1(\theta_1^*) - \mu^A \hat{v}_{x_1}^A = -\lambda^B \Delta_1^B. \tag{62}$$

Divide (61) by (62) and cross multiply to obtain

$$\Delta_1^B v_{x_2}^A G_2(\theta_2^*) + \Delta_1^B v_{x_2}^A \mu^A = \Delta_2^B v_{x_1}^A - \hat{v}_{x_1}^A \mu^A \Delta_2^B. \tag{63}$$

Divide through by $-\hat{v}_{x_1}^A \mu^A \Delta_2^B$ and rearrange to obtain

$$-\frac{v_{x_2}^A}{\hat{v}_{x_1}^A} \frac{\Delta_1^B}{\Delta_2^B} = 1 - \frac{v_{x_1}^A G_1(\theta_1^*)}{\hat{v}_{x_1}^A \mu^A} + \frac{\Delta_1^B}{\Delta_2^B} \frac{v_{x_2}^A G_2(\theta_2^*)}{\hat{v}_{x_1}^A \mu^A}. \tag{64}$$

Because the budget constraint is binding in region B, Δ_1^B and Δ_2^B have opposite signs, and the final term in (64) is negative. Hence,

$$-\frac{v_{x_2}^A}{\hat{v}_{x_1}^A} \frac{\Delta_1^B}{\Delta_2^B} < 1, \quad \text{and} \quad \frac{v_{x_2}^A}{\hat{v}_{x_1}^A} < -\frac{\Delta_2^B}{\Delta_1^B}. \tag{65}$$

Thus, we have shown that a slack budget constraint in region A implies (65). Thus, its negation, relation (27) of the text, precludes a slack budget constraint in region A. A similar argument shows that (28) is inconsistent with a slack budget constraint in region B. Thus, the conjunction of relations (27) and (28) implies that both parts of (25) bind. □

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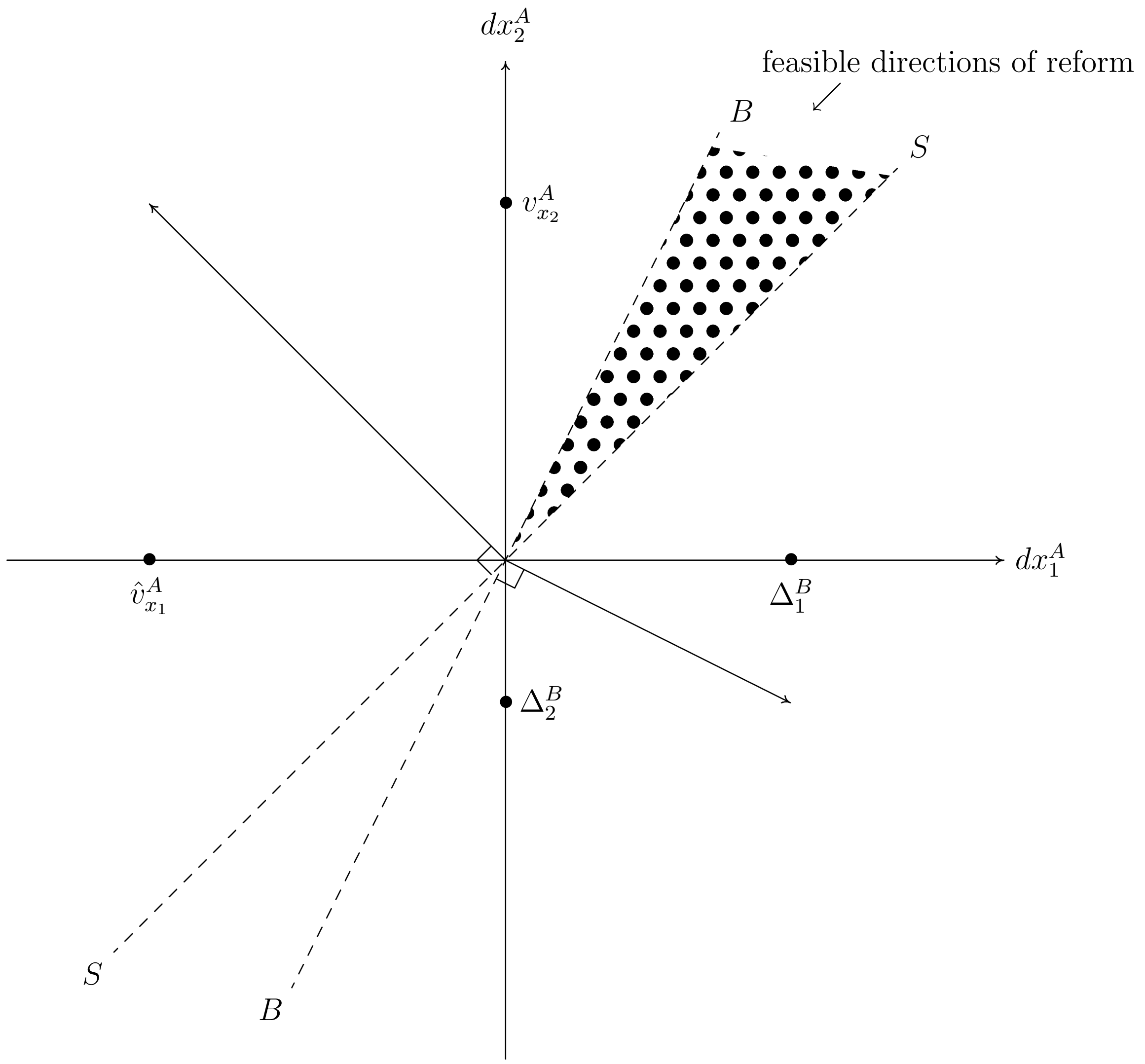

Figure 1. Feasible Redistributions of Surplus