MBA 405B

Problem set 1

Due: Monday, April 08, 2002, 5:00pm

1. There are three passengers on a ship sailing across the Atlantic who are carrying large sums of money. Suppose passenger A has $1000, passenger B has $1500 and passenger C has $2000. They each put their money in separate envelopes and give them to captain who puts them in a chest for safekeeping. One night the ship is caught in a storm and the chest is broken open. The captain picks up all the money the next day, but does not know much each of the three passengers gave him originally. He proposes the following solution. He asked each passenger to write down how much money they originally put into their envelope. They are to do this at the same time, without speaking to one another. If the numbers the agents write down add up to the total amount of money the captain picked up (assume he managed to find all $4500) then the captain gives each passenger what he claims he put in. Otherwise, the captain throws the money into ocean and the passengers get nothing.

a. Write this out as a formal one-shot game. Note that there are three players (the passengers) and that the captain is just a mediator who defines the strategy spaces and payoff functions of the agents.

I = {A,B,C};

S_i = {\$0 to infinity} (same for all passengers)

F_i = \$$s_i$ if $s_A + s_B + s_C = 4500$ (same for all passengers)
\$0 otherwise

b. Is truth telling a Nash equilibrium? Why or why not?

Yes. If the other two agents tell the truth, a truthful response by the remaining agent causes the sum of reports be 4500. In this case, agents get back the money they lost. Any other response by the remaining agent causes the sum to be something else and so the agents all get zero. Clearly, truth telling gives a higher payoff and is the only best response to truth telling.

c. Are there other Nash Equilibria?

Yes. Any set of reports that adds up to 4500 and are non-negative is Nash equilibrium. This is because the payoff to the agents is either zero or something positive. Deviating causes the sum to be different from 4500 giving zero payoffs. Thus, no deviation gives a higher payoff. There are also other Nash equilibria in which agents report a number higher than 4500. Here, the payoff is zero, but no deviation gives a higher payoff than zero either. Thus, these are weak Nash equilibria.

2. A Pirate ship has just captured a treasure chest in a daring raid. They find it contains 500 gold pieces. They are now faced with the problem of dividing the treasure between themselves. Suppose there are 10 pirates on board and we can order them from most, to least, ferocious. Thus, pirate number 1 is the most ferocious, and private number 10 is the most timid. The captain decides that he will let the crew vote over the division and forces the least ferocious pirate to make a proposal first. Pirates are a very rough group and they vote according to the following rules:

a. Voting is by majority rule. **In vote is tied, however, the proposal also passes**.
b. Pirates are selfish so each pirate will vote yes on a proposal only if he knows he is strictly better off than if he votes no.
c. Gold pieces cannot be divided. The proposal must be stated as the number of gold prices that each pirate is to receive.
d. The least ferocious pirate currently on board the ship must propose a division of the gold.
e. If a proposal is voted down, the pirate who made the proposal is tossed overboard, and the least ferocious pirate who remains on board proposes a new division of the gold.
f. This continues with successive proposals and being voted on until one is finally accepted.

Given these rules, and the fact that each pirate wants the most gold he can possibly get, what division should the least ferocious pirate (number 10) propose in the first round? Hint: You would work this out by backwards induction. What would happen if only one pirate remained? Given this what would happen if there were two pirates, then three, and so on?

One-pirate game: Pirate 1 proposes 500 for himself, votes "yes", and the "division" stands.

Two-pirate game: Pirate 2 proposes 500 for himself and 0 for pirate 1. Pirate 1 votes "no" Pirate 2 votes "yes". Since tie votes pass a proposal, the division stands.

Three-pirate game: Two votes are now needed to pass a proposal. Pirate 3 notices that if his proposal fails and he is tossed overboard, that in the two-pirate game that results, pirate 1 gets nothing. Thus he proposes that he (pirate 3) get 499, pirate 2 get 0, and pirate 1 gets 1. Since pirates vote "yes" only if they are better off than by voting "no", pirate 3 votes "yes", pirate 2 votes "no", and private 1 (who gets nothing in the two pirate game) votes "yes" (since he is being offered 1 gold priec which is more than 0).

Four-pirate game: Two votes are needed to pass a proposal. Pirate 4 notices that if his proposal fails and he is tossed overboard, that in the three-pirate game that results, pirate 2 gets nothing. Thus he proposes that he (pirate 4) get 499, pirate 3 gets 0, and pirate 2 gets 1, and pirate 1 gets 0. Since pirates vote "yes" only if they are better off than by voting "no", pirate 4 and 2 vote "yes", while pirate 3 and 1 vote no. The proposal passes.

You can verify that by building this up to the ten-pirate game, we get the following payoff: (496, 0, 1, 0, 1, 0, 1, 0, 1, 0) where the payoffs are listed from pirate 10 (who gets the 496) down to pirate 1 (who gets 0).

3. Consider the following game between Arthur Andersen (row player, left payoff) and the Enron (column player, right payoff). Please solve for the equilibrium by iterated removal of dominated strategies.

Enron/Andersen	Coordinate cover-up with Andersen	Let Anderson cover-up, but CYA	Tell stockholders about problems
Coordinate cover-up with Enron	2, 100	1, 150	0, 1
Let Enron cover-up, but CYA	10, 20	2, 30	1, 2
Report irregularities to the government	5, 0	4, 2	2, 3

4. Consider the sequential game below. Using the rule of "look forward and reason backward", also called backwards induction, find the subgame perfect equilibrium. Briefly explain your reasoning.