

## MBA 405B

### Handout for Lecture 7

This lecture is an introduction to the problem of incentives in several contexts.

#### Vickery Auctions, Public Goods, and Contracts.

Public goods have the property of **nonrivalry** in consumption. For example, a radio broadcast is a public good since everyone in town can listen to same broadcast, and the fact that one person listens in does not affect the amount of broadcast available to anyone else. In other words, each person consumes the *total* amount that is produced. A private good, in contrast has **pure rivalry** in consumption. For example, each bite I take of an apple means that the amount of apple available for anyone else to consume is reduced by exactly one bite.

Recall the story of the three little pigs. In the sequel to the famous children's story, they are considering whether or not they should build a build wolf siren. The wolf siren is a public good that would warn all of the brothers that they should run and hid in the brick house. Thus, each pig gets benefits from hearing the warning (nonrivalry), although not necessarily the same benefit.

The problem is that when they get together to fund this public project, they are each tempted to under report the benefit they expect to receive in order to lower the share of the cost they each have to pay. (This is called **free-riding**) If they all do this, however, the reported benefits are lower then the costs and based on this, they would end up not building the project. This is true despite the fact that if they all told the truth, they would find that the benefits are higher than the costs and they would all be better off as a result. So the issue is: how can we get honest revelation of the benefit to each pig in order to decide whether or not to build the siren, and if so, how to share the costs?

Suppose the true costs and benefits are the following (the subscripts stand for brick, wood and straw, what each pig used to build his house):

$$B_b = 100 \quad B_w = 150 \quad B_s = 200 \quad C = 400$$

Notice that the total benefits are 450, which exceeds the costs.

We propose the following mechanism, which is a version of the famous Vickery auction. Each pig is asked to report his benefit. Call these reports  $R_i$ , and note that these need not be truthful. If the reported benefits add up to less than the cost, then siren is not built. If

the reported benefits add up to more than the costs, then each pig has to pay and tax ( $T_i$ ) equal to the residual net benefit not accounted for by the reports of the other pigs. To understand this, consider this from the standpoint of the pig that build out of straw. The taxes he has to pay are the following:

$$\begin{aligned} T_s &= 400 - R_b - R_w & \text{if} & \quad R_b + R_w + R_s \geq 400 \\ T_s &= 0 & \text{if} & \quad R_b + R_w + R_s < 400 \end{aligned}$$

Suppose the other pigs tell the truth. What should the straw pig do?

Benefits of telling the truth: In this case  $R_s = B_s$  which implies a tax of:

$$400 - 100 - 150 = 150$$

and a net payoff of:

$$200 - 150 = 50.$$

Benefits of lying: If the straw pig reports any number over 100, the siren is built and the tax and net payoff are the same as in the truth telling case. Thus, the amount of tax does not depend on the pig's report provided the siren is still built. On the other hand, if the pig gives a report of less than 100, the reports sum to less than the cost, and siren is not built and both taxes and net benefits are zero. This costs the pig a surplus of 50. Therefore, telling the truth is a best response.

If fact, telling the true is not only Nash equilibrium, it is dominant strategy equilibrium. Notice that the argument above applies just as well if the reports of the other pigs are not truthful. There is never any gain and there are potential losses from giving false reports.

This also applies if the public good is too costly to build given the benefits. In this case a false report might cause the project to be built and the taxes to each pig has to pay being higher than the benefits they receive.

This looks great as it gets truth telling in dominate strategy and causes the public good to be build if and only if the benefits exceed the costs. A significant problem with this mechanism is budget balance. Add up the total revenue in the case of truth telling:

$$T_s = 400 - R_b - R_w = 400 - 100 - 150 = 150$$

$$T_w = 400 - R_b - R_s = 400 - 100 - 200 = 100$$

$$T_b = 400 - R_w - R_s = 400 - 150 - 200 = 50$$

The total is 300, which is less than the cost. In general the budget will not balance.

Conclusion. What makes this mechanism work is that making an honest report can never hurt an agent. This is because an agent's share of the taxes and therefore his net payoff does not depend on his report. This is the key insight.

Let's use this idea to develop a contract between two partners who have private information. Suppose they are going to go in on a joint venture to build a restaurant. The Partner A will undertake to find a site and acquire the necessary land and permits to build a new restaurant. The partner B will undertake to build the restaurant and get it staffed. They expect to make a total of 5 million out of the joint venture and they have to commit to work together months before the project can begin. The problem is they don't know how much it will cost of do their shares of the work, and won't know until they get into the project.

Partner A thinks that getting the site together will cost either 1 or 2 million and each is equally likely.

Partner B thinks it will cost either 2 or 4 million to build and staff the restaurant and each is equally likely.

They agree to the following contract: If the costs turn out to total 6 million, they will not build the restaurant. Otherwise, they will go ahead and complete the project with partner A getting 1.5 million and Partner B getting 3.5 million of the revenue.

Notice that in some cases they will want to renege on the contract. To prevent this, each agrees to pay a penalty of  $\frac{3}{4}$  million to be allowed to walk away from the partnership. Notice this is enough to prevent them from backing out since they only lose  $\frac{1}{2}$  million if the costs are high, and they have to follow through anyway.

Each partner does an expected value calculation assuming truth telling:

If both costs are low, he gets .5

If his costs are low and the others are high, he gets .5

If his costs are high and the others are low, he gets -.5

If both costs are high, they don't build the project, and he gets nothing.

Each of these is equally likely, so the expected profit is .125 million.

Thus, *ex anti*, they will sign the contract. Will they tell the truth? If a partner has low costs, reporting this assures the project will be build and he will get .5 million. Thus, they should certainly reveal this. If a partner has high costs, then the only time this makes a difference in payoff is if the other partner also reports high costs, in which case the project is abandoned. Since this is what a high cost partner wants, he should certainly report this.

Notice two things:

- The possibility of abandoning the partnership if it is not profitable for anyone is what makes this a profitable partnership in the first place. In other words, the fact

- that you can back out in certain circumstances, makes you more likely to enter it in the first place. (What are the implications for marriage?)
- Although this is profitable *ex anti*, it may not be for each partner *ex post*. Thus it is not *ex post* individually rational.

## Auctions

Auctions come in many forms.

In a *first price ascending bid auction*, bidder bid in turn and highest bidder wins and pays what he offered. One problem is that the bidders with the highest value will not have to bid more than just above the price of the bidder with the second highest value. Thus, the maximum price may not be obtained.

In *first price sealed bid auction*, bidding is not public. We can think of bidders as placing his highest offer in an envelope. This is often used to award contracts for public projects (in this case, the lowest bidder wins.) The hope is that since bidder with the highest value does not get to see the second highest bid, he may bid closer to his true value. In fact, the strategies for the bidders are quite hard to calculate and depend critically on what he bidders guess about each other. Since I don't know the values of the other bidders, I might try to shade my bid and if everyone does this, the price may end up being lower. This is just as plausible as the case in which bidders overestimate the values of one another and so bid very close to their values (which produces a selling price higher than in the open auction). The outcome is not clear. A secondary problem is that if the bidder with the highest value estimates that the next highest bidders value to be lower than they are in fact, while the second highest bidder thinks that the other bidders have high values, the second highest value bidder might win the auction. Thus, the good can be misallocated.

This is especially troubling in the cases where it is important that the highest value bidder wins, An example is spectrum auctions. Here, they use a second price auction.

In a second *price sealed bid auction*, the highest bidder wins, but pays only the second highest bid. This implies that there is a dominant strategy to bid your true value. To assume you bid the truth and note the following.

1. If you bid your value and win, you never pay more than the item is worth to you.
2. If bid your value and lose, the only way you could have won would be to pay the winning bid, which has to be above your value in this case.
3. If you win the auction, raising your bid does not affect the price that you pay (since you pay the second price). If you lower your bid too much, you end up losing the auction, which makes you worse off
4. If you lose the auction, raising your bid (above your true value) might cause you to win and pay the old winning bid (which must be above your true value).

How do you choose which auction to use? It depends on whether maximizing revenue or getting the efficient allocation most important.

Other considerations:

- Does the number of bidders make a difference?
- Does it matter if this is a private value (each person values a painting differently) or common value (Drilling rights have the same value to all firms, but they are uncertain) auction.
- What happens if there are multiple objects? Bidding of the early objects may reveal something about the private value you place on future objects. This may work to the advantage or disadvantage of the bidders or the seller.
- What if there are multiple objects that become more valuable in groups. For example, owning the spectrum rights to have a cell phone network over the entire country is more than twice as valuable than having the rights to half the country.

### **Principal-Agent problem**

A broad class of incentive problems is called “principal-agent” problems. The agent in this case is someone responsible to carry out an action on behalf of the principal.

Consider a Boss-Worker relationship. The reason that this is a “Principal-agent problem” is that worker has different objectives than the boss, and so may not always do exactly what the boss wants. Even worse, it may be difficult for the boss to verify the worker’s performance.

Other examples:

Parents-Teachers

Voters-Politicians

Generals-Soldiers

Companies-Consultants

Stockholders-Boards of Directors

Boards of Directors-CEO’s

What is the broad solution? Since the problem is that the agents have interests that diverge from the principal’s, *the solution is to make the agent share the same objectives as the principle through an incentive scheme.*

Example 1: We want our teachers to educate our children and keep them safe. Teachers want high pay, not to work too hard, to get promoted, not to get in trouble, and to an extent, do a good job (this is like altruism). If we simply pay teachers on a seniority basis, there is

little reason to work hard or do a good job other than to the extent a teacher enjoys his work. A teacher wants to avoid getting fired, so he will probably not show up drunk or molest students, but this is all this seniority pay system assures.

Suppose instead we give teachers merit pay for the scores that the kids get on a standardized test. The higher the reward per test point, the harder teachers work on getting these high scores. Is this a good thing? If the tests are good at measuring the outcome we want, then yes. But it is hard to design a good test. It may be possible to “teach to the test” and get artificially high scores. If the test is not comprehensive, the teacher may neglect other important things that are not tested.

What if we reward teachers for the fraction of students that graduate or go to college. Then the teacher is best served by focusing only on the borderline students. The smart ones will go on anyway, and the dumb ones won't so both are a waste of time. This is not the allocation of effort we want.

The bottom line is that although the objective is to get the objectives of the principal and the agent to be identical, this is quite difficult in practice. Writing contracts that produce the correct incentive structure is a complicated problem.

Example 2: How should your teaching evaluations be factored into my pay?

---

**Final exam:**  
**Wednesday, May 8, 1:30-4:30p 241 WH and 243 WH**

---