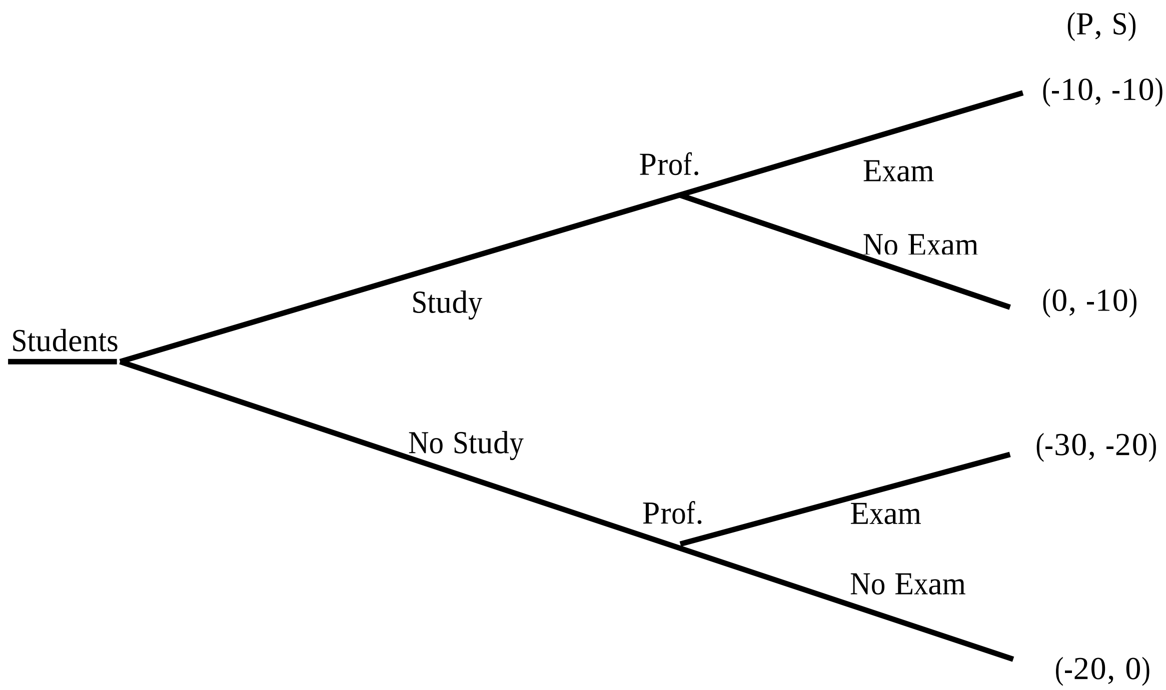
MBA 405B

Handout for Lecture 2

In the previous lecture finished we discussed sequential interaction as opposed to simultaneous interaction.

One thing we observed in the game tree we used to describe a sequential game is that there can be a large number of equilibria, more than in the analogous normal form game Consider the exam game again:



Here are all the possible strategy combinations for this game. What are the equilibria?

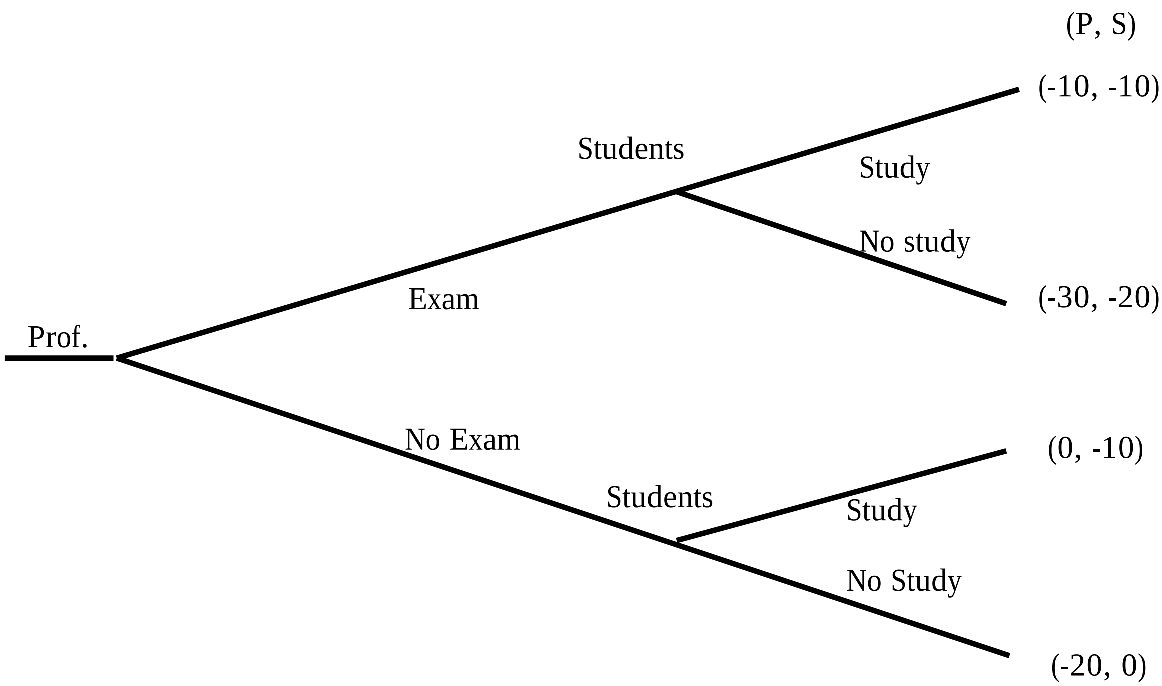
1. [(NS→NE, S→NE); NS]
2. [(NS→E, S→NE); NS]
3. [(NS→NE, S→E); NS]
4. [(NS→E, S→E); NS]
5. [(NS→NE, S→NE); S]
6. [(NS→E, S→NE); S]
7. [(NS→NE, S→E); S]
8. [(NS→E, S→E); S]

What can we say about narrowing the set of equilibria? Which equilibria are credible? One thing we have here we did not have in the normal form game is a sequence of play. This creates the notion of a *subgame*. There is a subgame beginning at each decision node of the game tree. A reasonable condition to impose is that all the agents are behaving rationally in every subgame. Thus:

A set of strategies (one for each player) is said to be *subgame perfect* if every player is using a best response in every subgame (including those subgames which may not be seen in equilibrium).

Which of the Nash equilibria above are subgame perfect equilibria (SGE) for exam game?

What happens if we change this game to have the professor move first?



This leads us to our first rule:

Rule 1: Look ahead and reason back

In game theory, this is called *backward induction.* For this to work, two things must be true:

- You must be able to observe your opponent's moves and so be able to condition on them
- There must be irreversible commitment.

Consider the classic cake eating game.

Two players: Ali and Baba who must divide a cake.

One round game: Ali proposes a division, which Baba must accept or reject. If he rejects the cake disappears. This is sometimes called an ultimatum game.

Two round game: Ali proposes a division which Baba must accept or reject. If he rejects, half the cake evaporates and then Baba proposes a division to Ali. If Ali rejects, the cake disappears.

N-round game: Ali proposes a division, which Baba must accept or reject. Each time there is a rejection, half the remaining cake evaporates and the other player proposes a division. After N-rounds of play, the cake disappears.

- What is the equilibrium of the one and two round games?
- What happens if less than half the cake disappears each time?
- What is the relationship of this game to discounting?
- What if one of the agents is less patient than the other.

To rule one, we can also now add another principle from that comes out of the first lecture.

Rule 2: If you have a dominant strategy, use it.

Clearly, if a strategy yields the highest possible payoff in every situation, then there is no question that one should employ it.

Notice that the principle of reasoning backwards allowed us to reduce the set of equilibria by removing strategies that were not credible (not SPE). In essence, we reject the use of strategies in sequential games that require agents do things that would against their own interests. Can we find a similar type of logic for normal form games?

This brings us the idea of *dominated strategies*. In a normal form games, a strategy is said to be dominated there is another strategy that yields a higher payoff regardless of the actions of the other players. More formally:

$s_i \in S_i$ is a dominated strategy if for some $s_i' \in S_i$ and all $s_{-i} \in S_{-i}$

$$F_i(s_i, s_{-i}) \leq F_i(s_i', s_{-i}).$$

(Note, this is a definition of a weakly dominated strategy. The definition of a strongly dominated strategy is the same except that the inequality is strict.)

For example, suppose that two firms are considering a merger. Independently of this, they can choose to cooperate in developing and deploying a new piece of management software. In agreeing to merge or cooperate a firm incurs costs that are nonrecoverable if the other firm does match its strategy. The other firm may also receive some competitive advantage in declining an offer of merger or cooperation since the offer contains useful private information about the other firm. Here is the game they play:

Merger game	Agree to merge and cooperate	Agree to merge but not cooperate	Disagree but cooperate	Disagree and don't cooperate
Agree to merge and cooperate	100 100	70 90	10 30	2 8
Agree to merge but not cooperate	90 70	50 50	10 20	5 7
Disagree but cooperate	30 10	20 10	10 10	7 6
Disagree and don't cooperate	8 3	7 5	6 7	0 0

- What are the Nash equilibria?
- What happens if we remove dominated strategies?

Thus, iterated removal of dominated strategies has helped us select a Nash equilibrium.

Consider the problem facing a two football teams trying to choose strategies. The offensive team wants to make the most yardage and the defensive team want to see the least yardage gained:

Football Game	Counter run	Counter Pass	Blitz
Run	3	7	15
Pass	9	8	10

- Is there a Nash equilibrium?
- Are there dominant strategies?
- What happens if we remove dominated strategies?

This gives us our next rule:

Rule 3: Remove dominated strategies from consideration and continue to do so successively.

What if none of the first three rules work? Then we have to work to find the Nash equilibrium of the game. Sometimes this is by inspection. Other times we can calculate them.

For example: Consider two duopolists competing for profits. Suppose the following conditions hold:

<u>Demand</u>

$$P(Q) = 1000\text{-}Q$$

<u>Cost</u>

$$TC_1(Q_1) = TC_2(Q_2) = Q_i^2$$

Let's take firm 2's actions as given and calculate firm 1's best response. To do this, we have to maximize firm 1's profits

<u>Profits</u>

$$\Pi_1 = Q_1(1000 - Q_2 - Q_1) - Q_i^2$$

Take the derivative and set this equal to zero:

$$(1000 - Q_2 - Q_1) - Q_1 - 2Q_1 = 0$$

$$1000 - Q_2 - 4Q_1 = 0$$

$$Q_1 = 250 - Q_2/4$$

This is the best response function for firm 1. Since the firms are symmetric, there is a similar best response for firm 2:

$$Q_2 = 250 – Q_1/4$$

Since both firms must be following a best response at a Nash equilibrium, both equations have to be satisfied. Thus, we have two equations and two unknowns. Let us substitute firm 2's best response into firm 1's.

$$Q_1 = 250 - (250 - Q_1/4)/4$$

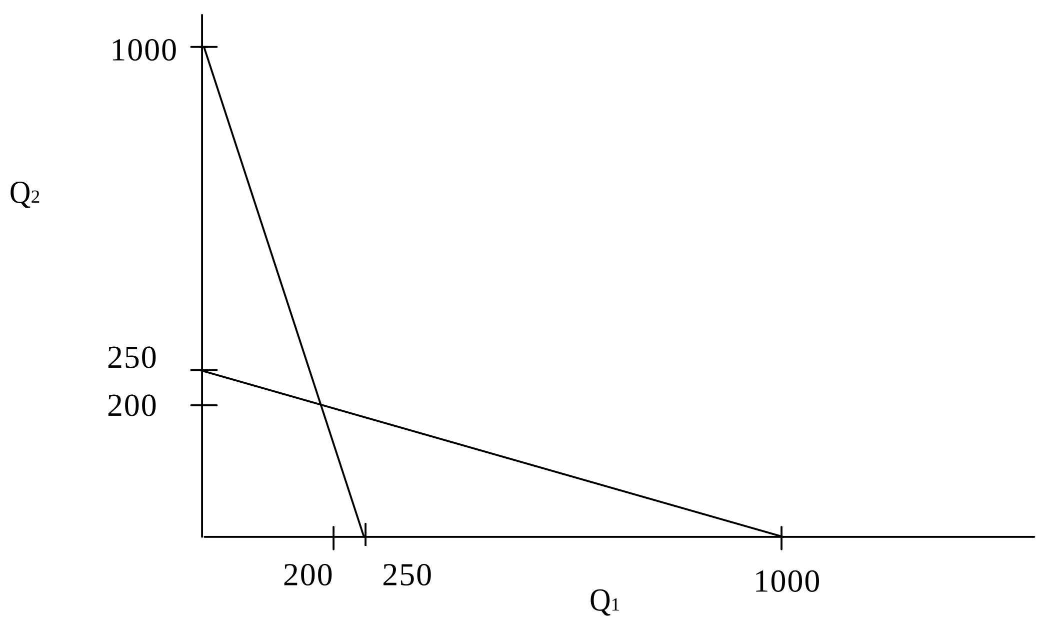
$$4Q_1 = 1000 - (250 - Q_1/4)$$

$$4Q_1 = 750 + Q_1/4$$

$$3.75Q_1 = 750$$

$$Q_1 = 200$$

Thus, each firm producing 200 units is a fixed point. And is a Nash equilibrium Graphically:



This gives us our last rule:

Rule 4: Having removed the simple avenue of looking for dominate strategies or ruling out dominated ones, the next thing to do is to look for an equilibrium of the game.

This can be hard, and we will spend more time discussing how to find equilibria.