MBA 405B

Handout for Lecture 1

Games and markets

Economics uses two main tools to analyze the world. The first revolves around markets. The focus is to study agents who, on the average behave in rational or at least predictable ways. The key to this analysis is the assumption that agents are small relative to the market and therefore take their environment (prices, for example) as fixed and not liable to influenced by their own market choices.

The second is game theory. In contrast to markets economics, game theory focuses on interactions between small groups of agents. These agents must take into account that their actions may affect the strategic choices of the other agents, and therefore, their own payoffs. Game theory is especially good at modeling situations with incomplete or asymmetric information, agents who may be less than fully rational, and in which agents' beliefs and expectations about the environment and other agents' behaviors may affect strategic choices. Although games with many agents can also be treated, as games become larger, the outcomes tend to look more and more like those that come from markets.

Motivating examples

- Should I donate to PBS?
- Should I speed on the way to Chicago?
- Should I cheat on my income tax?
- Should I bid my true reservation price on Ebay?

Normal form games

A normal form game is a simultaneous move, one-shot game. Games don't literally have to take place simultaneously. It just has to be that each agent must commit to a strategy before he knows the other agents' strategies.

Three basic elements are needed to define a *normal form game*:

- Players
- Strategies
- Payoff function

For example.

Consider a game between and students working together on a problem set that will be graded on a curve. Each student knows some of the answers and can either reveal or not reveal his knowledge. This will affect the score on they each get on the problem set. We can write this is what is called “bimatrix form” as follows. The lower left-hand number is the row player’s payoff, and the upper right-hand number is the column player’s payoff.

MBA Team Game		Ray	
		Reveal your answers	Don’t reveal your answers
Bob	Reveal your answers	B B	A D
	Don’t reveal your answers	D A	C C

Formally, we can write this game as follows:

Players:
$i \in (1, 2, \ldots, I) \equiv \mathbf{I}$
In this case $\mathbf{I} \equiv \{\text{Bob, Ray}\}$

Strategies:
$s_i \in S_i$
$s = (s_1, \ldots, s_I) \in S \equiv S_1 \times S_2 \times \ldots \times S_I$
$s_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_I) \in S_{-i}$
In this case: $S_b = S_r \equiv$ {Reveal, Don’t Reveal} $\equiv$ {R. DR}

Payoffs:
$F : S \rightarrow P^I$ where P^I is an I-dimensional payoff space, possibly a Euclidian space, but necessarily.
$F \equiv (F_1, \ldots, F_I)$ Where $F_i : S \rightarrow P$.
In this case:

$F_b(R, R) = B$ $F_r(R, R) = B$
$F_b(R, DR) = D$ $F_r(R, DR) = A$
$F_b(DR, R) = A$ $F_r(DR, R) = D$
$F_b(DR, DR) = C$ $F_r(DR, DR) = C$

Note, the first entry is Bob’s strategy and the second is Ray’s strategy.

Nash equilibrium

The next question is: how do we figure out what is likely to happen in this game? In game theory, there are several notions of equilibrium. The most important is Nash equilibrium (NE).

Informally, in a *Nash equilibrium*, all agents choose a strategy that is a best response to the ones chosen by the other agents in the game.

More formally:

$s \in S$ is a Nash equilibrium if for all $i \in \mathbf{I}$ and all $s_i' \in S_i$

$$F_i(s_i, s_{-i}) \geq F_i(s_i', s_{-i})$$

What is the NE of the MBA Team Game?

Dominant strategy equilibrium

A second very important type of equilibrium is called *dominant strategy equilibrium* (DSE).

Informally, a strategy is dominant for a player if it yields him the highest possible payoff in every given situation. If all players have a dominant strategy in a game, this forms a DSE.

More formally:

$s_i \in S_i$ is a *dominant strategy* if for all $s_i' \in S_i$ and all $s_{-i} \in S_{-i}$

$$F_i(s_i, s_{-i}) \geq F_i(s_i', s_{-i})$$

$s \in S$ is a DSE if for all $i \in \mathbf{I}$, $s_i \in S_i$ is a dominant strategy.

What is the DSE of the MBA Team Game?

An Example

Consider a game between professors and students. Students can study or not study and professors can give or not give exams. Students hate to study (-10) but hate to get bad grades as well (-20). Professors hate to give exams (-10) but also hate it when students don't study (-20). We can write this is what is called bimatrix form below. The lower left if the professor's payoff and the upper right is though student's payoff.

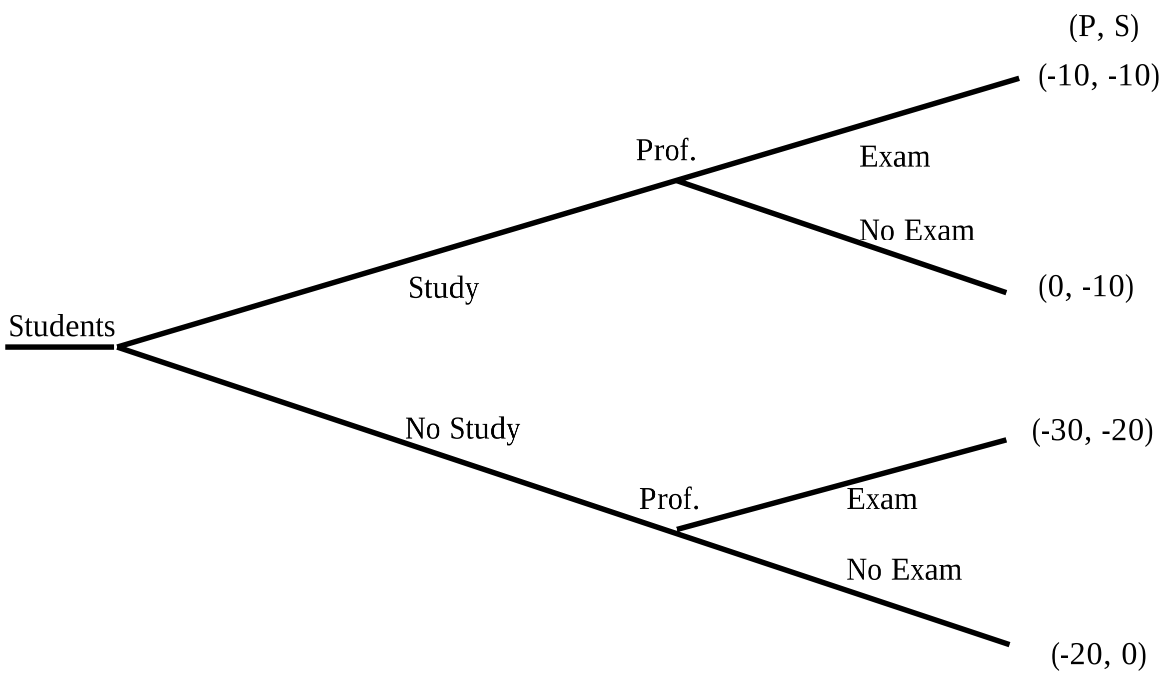
Exam Game		Students	
		Study for exam	Don't study for exam
Professors	Give exam	-10 -10	-20 -30
	Don't give exam	-10 0	0 -20

What are the NE and DSE of these games? What do we see in real life. Why?

We will cover these issues in more detail latter in the course

Extensive form games

One complication is that many games are be played sequentially. For example, one player may move first, and then be followed by a second player who gets to see the move the first play made before deciding on his own strategy.



Suppose that the exam game was played sequentially. The students decide to choose to study or not first, and seeing how hard this student are working, the professor decides to give or not give an exam. Note that the professor's strategy is contingent on the student's decision. We specify these contingent strategies as follows:

$$s_{-i} \rightarrow s_i$$

Consider the professor's strategy:

(NS→E, S→NE)

What is the student's best response? Is the professor's best strategy a best response to this, and so is this Nash equilibrium?

What about the pair [(NS→NE, S→NE) ; NS]? Is this a NE? Which do find more credible?