Beneficial Inequality in the Provision of Municipal Services:†
Why Rich Neighborhoods Should Get Plowed First.

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Abstract

This paper provides an explanation for the common observation that higher income neighborhoods typically receive better public services than lower income neighborhoods. Intuitively, one might expect that lower income groups, which usually form the voting majority of cities, would object to an unfair allocation of this nature. Wealthy individuals, however, have the option of moving to the suburbs. As we learn from the tax competition literature, mobile factors are generally able to command a premium. Since institutional constraints prevent regressive taxation and public goods are by definition consumed in equal quantity by all agents, only public services remain as an instrument for municipalities to use to keep wealthy agents in their tax base. We show that both rich and poor agents benefit from this differential access to public services and explore how factors like the ratio of rich to poor and the differences between their incomes affect the equilibrium allocation.

1. Introduction

Local governments play an important role in the day-to-day life of Americans. They provide education, garbage collection, police protection, and many other fundamental services that the population cannot do without. This is especially true for the United States, whose population is largely urbanized.

A large literature in the tradition of Tiebout (1956) has developed, which has explored the possibility that competition between jurisdictions may induce efficient provisions of these goods and services. Much less attention has been devoted to the question of how localities choose to allocate goods internally. In an important early paper that considered this issue, Inman and Rubinfeld (1979) observed that the typical city shows potentially significant inequalities in the provision of municipal services across neighborhoods. There are a few neighborhoods with low crime rates, green parks, good schools and clean streets, but most have occasional crime, decent streets, and schools financed at an adequate level. Of course, there are always some parts of a city with high crime rates, dirty and potholed streets and poor schools. Putting this in quantitative terms, Inman and Rubinfeld determine that families with $10,000 yearly incomes receive on average 25% more in services than families with $5,000 incomes, while a family with a $20,000 yearly income receives an additional 25% more than the $10,000 income family.

This inequality is a bit mysterious if cities are ruled democratically. Intuitively, one might expect that lower income groups would not willingly tolerate an unfair allocation such as this. Of course, one could explain this by telling a political-economy story of how political donations, low voter turnout amongst the poor, or ignorance and apathy effectively disenfranchise the poor. The main point of this paper is to provide an explanation for this phenomenon that does not rely on this type of argument. We will show that in fact, the poor often have an interest in supporting this type of inequality and are better off as a result. As a secondary point, we will also explore how the parameters of the economy affect the relative treatment of the rich and poor by local governments.

We consider a model with a private good, a public good, and a public service (a publicly provided private good). There are two types of agents, rich and poor, who differ in income but have identical tastes. We assume the poor live in a jurisdiction called "the city" and form the majority of the population. The rich can choose to live in the city as well, but may instead choose to live in "the suburbs" apart from the poor. We assume that because of zoning, commuting costs, or some other unmodeled factor, the poor are unable to move to the suburbs. Public goods and services are funded by a proportional income tax, and public goods are purely nonrival. The proportionality of this income tax (which may roughly proxy for sales taxes or property taxes) is a key modeling assumption. It precludes the city from using the tax system to discriminate either in favor of or against the rich.

As we suggest above, there may be circumstances in which it is in the interest of the poor to treat the rich better than they treat themselves with respect to municipal services. Intuitively, what drives this is that the poor would prefer that the rich live in the city as this allows them to capture their income as part of the tax base. The rich also potentially benefit from living in the city, since the larger population (the rich plus the poor) allows higher public goods production at lower per capita cost. If there were nothing standing their way, the poor would choose to impose very high income taxes on everyone and redistribute the resulting revenue in the form of high levels of public services targeted specifically at the poor. The rich, however, have the option of moving to the suburbs. To prevent this, the poor must make the rich at least as well off in the city as they would be on their own. Since tax rates and public goods consumption are the same by constraint, the only instrument the poor have available to provide the rich enough benefit to prevent their migration is public services.[1] Thus, the poor may choose to give the rich extra public services to induce them to stay in the city and pay city taxes. The rich are at least as well off as they would be in the suburbs by constraint, and the poor are strictly better off. Thus, the poor should support such a

[1] Of course, if the rich and poor have different tastes, providing public goods that the rich especially enjoy (golf courses, opera) may be a backdoor way of differentially providing benefits to the rich despite the nonrivalry of the goods in question.

differential allocation and it results in a Pareto superior social allocation.

We are, of course, not the first to look at the differential provision of public services within a jurisdiction. In addition to the work of Inman and Rubinfeld, related papers include Gramlich and Rubinfeld (1982, and references therein), Behrman and Craig (1987), Craig (1987), Craig and Heikkila (1989), Schwartz (1993). These last models look at the empirical distribution of a certain public output (for example, "safety" within a city) by examining the local government allocation of an input (the input for safety being "police"). For a very interesting application of a model quite close to the one proposed in this paper see Rothstein and Te (2001), who consider a case in which the production costs are lower for the rich and thus the poor have a relatively harder time attracting them into an integrated city.

Most closely related to our model is the work of Craig and Holsey (1997) and Hoyt and Lee (1998). Craig and Holsey develop a framework that determines the efficient distribution of a publicly provided input within one jurisdiction, taking into account the congestion effects that may be present. They show that the efficient distribution of publicly provided inputs depends on the individual tastes of each local resident, and on each individual's ability to convert publicly provided inputs into public services output.[2]

Hoyt and Lee take a somewhat different approach. In a framework with rich and poor agents, they develop a model that shows how fiscal zoning regulations can play a role in improving the welfare of a community. If such a policy tool is not available to the community (perhaps because it is illegal), the authors show that an alternative policy with similar outcome (to sort out the poor) is for the rich community to over-provide a luxury private good (for example, a golf course). The provision of such high-income elasticity goods will make the jurisdiction less attractive to the poor, and so make the communities more homogeneous. Notice that Hoyt and Lee focus their attention on how

[2] We are assuming that spending on public goods and services generates outputs that are valued by consumers. We should acknowledge, however, that some empirical literature has suggested that public outputs (school quality in particular) may not be closely tied to the level of spending input (see Hanushek 1986, for example).

to keep the two income groups separate. In contrast, our model identifies conditions under which it is possible for the rich and poor to benefit from living together.

There is also a relationship between our analysis and the tax competition literature. Seminal papers in this line of research include Mintz and Tulkens (1986), Wilson (1986), Wildasin (1988) and Bucovetsky (1991). More recent extensions of the literature can be found in Bucovetsky (1995), Henderson (1994, 1995), Lee (1997) and Hindriks (1998). See Wilson (1999) and Wildasin (2003)[3] for surveys on tax competition. In such models, jurisdictions choose their tax rates optimally taking capital flows and their net-return impact into account, while viewing the tax rates chosen by other jurisdictions as parametric. In our model, the rich are the mobile factor instead of capital, and public services are the instrument used to attract the factor rather than tax rates. Nevertheless, the basic insight, that the outside offers available to the mobile factor allow these factors to force jurisdictions to use the instruments available to induce them to move to the locality, is the same.

The paper proceeds as follows. The next section introduces the general model. In section 3, we solve for the equilibrium in the general case and give some results. In section 4, we develop a more specific example of our model and provide additional results. Section 5 concludes the paper.

2. The Model

We consider a model with a private consumption good x, a local public good y, and public services s. Public services in this case are purely rival private goods that are provided exclusively by the government. We assume that the local public good is a non-excludable and non-rival good also provided by the city government.

We will analyze an economy with two types of people, the rich and the poor,

[3] Wildasin's paper is included in a special issue of the *Journal of Public Economic Theory* on tax competition.

which we will distinguish by the superscripts r and p respectively on utility functions, and as subscripts elsewhere. We shall assume that rich and poor share the same preferences and differ only in their initial endowments of private goods. We denote these endowments ω_r and ω_p, with $\omega_r > \omega_p$. Thus:

$$U^r(x,s,y) = U^p(x,s,y) = U(x,s,y) \quad \text{and} \quad \omega_r > \omega_p > 0.$$

This utility function is assumed to be continuously differentiable, strongly monotonic, and strongly quasi-concave. There are N_p poor, and N_r rich agents in the population. We shall also assume that there are more poor agents than rich ones. Thus:

$$N_p > N_r.$$

The poor always live in a location we call "the city". The rich may either live in the city or move to different location called "the suburbs". In the interest of simplicity, we leave unmodeled the factors that prevent the poor from following the rich. A more complete treatment would have to include zoning or show how the lack of public goods in the suburbs prevents the poor from moving in with the rich. See Glaeser (1998) for a discussion of these issues. Including these factors would change the details of our results, but not the substance.

We assume that technology is linear and that

$$C_s \text{ and } C_y$$

are the per-unit cost of public services and public good, respectively, in terms of the private good.[4] Production of both of these goods is funded by a proportional tax on endowments. This might proxy for a flat state income tax or a sales tax, for example.[5]

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[4] Note in particular that this contrasts with Rothstein and Te (2001) who work with production costs that might be different for the rich and poor.

[5] We are, of course, aware that typically cities in the United States do not impose (proportional) income taxes, although there are exceptions like New York City (which has its own income tax). The purpose of modeling taxation in this way is to proxy city sales or property taxes. Sales taxes are largely proportional (even though some goods may carry a lower sales tax, but as a first approximation pro-

The tax necessary to pay for public production will be treated as determined by the production choices and therefore is not defined as a separate choice variable of the localities.

3. Equilibrium in the Suburban Migration Game

The problem faced by the poor is the following. The rich always have the option of migrating to the suburbs. If they do so, they would obviously choose the socially optimal levels of public goods and services for themselves. This implies a reservation utility for the rich. The poor may choose to accept that the higher income group will live in the suburbs taking their taxable endowments with them. In this case, the poor choose the socially optimal levels of public goods and services as a city consisting only of themselves. Alternatively, they may choose to try to entice the rich to join them. Doing so requires that the poor meet the reservation utility of the rich.

There are two ways that the poor can do this. First, the city has a larger population than the suburbs, especially if the rich move in. This allows them to spread the cost of public goods more widely and provide higher levels (or better quality) at lower per capita cost. Note that it is the presence of public goods that implies that it is socially efficient for the rich and poor to live together. Indeed, one of the major reasons the rich choose to live in cities is the access that it gives them to amenities such as parks, museums, the opera, etc. If public services were produced under increasing returns to scale, we would see the same effect even without public goods.[6]

The other way is to provide the rich with higher levels of public services than the poor enjoy. This may mean more frequent police patrols, better schools and street

portionality does hold). And property taxes do increase with the value of the house agents own; if we reasonably assume that richer agents live in a more valuable house (which carries a higher assessment in terms of property tax), proportionality of taxation seems to be again a reasonable assumption.

[6] Thanks to Jonathan Hamilton for pointing this out.

lighting, quicker and more frequent snow removal, etc.[7]

From a game theoretic standpoint, we consider a sequential game in which the poor move first by making an offer to rich and the rich subsequently either accept and move to the city or reject and live in the suburbs. We consider only the subgame perfect equilibrium, and so the poor know the rich will accept any offer that meets their reservation utility. This means that the poor can perfectly anticipate the strategic choices of the rich by backwards induction, and so the game becomes trivial. In the interest in simplifying notation, we have therefore chosen not to define the game more formally in the following. Instead, we begin by considering the three possible types of localities we might see in equilibrium: a segregated city, a segregated suburb and an integrated city.

Segregated City (SC)

When the poor accept that the rich are going to live in the suburbs, they are left with the problem of maximizing per capita utility alone. Formally, this problem is as follows:

$$\max_{x_p, s_p, y_p} U^p(x_p, s_p, y_p)$$

$$\text{s.t. } N_p x_p + C_s N_p s_p + C_y y_p = N_p \omega_p$$

Note that it is immediate that this implies the following Samuelson condition:

$$N_p \frac{U^p_{y_p}}{U^p_{x_p}} = C_y$$

where we use the convention that a U_z is the derivative of U with respect to z. Denote the solution to this problem as $(x_p^{SC}, s_p^{SC}, y_p^{SC})$.

Segregated Suburb (SS)

[7] Obviously there is a partial non-rivalry aspect to most of these examples. Notice, however, that (1) the cost of providing these goods is more or less linear in the number of agents served, and (2) it is possible to provide different neighborhoods different levels of these goods. Thus, modeling them as purely rival public services appears to be a reasonable approximation.

If the rich choose to migrate to suburbs, then they face a problem that is essentially identical to the one of the poor above:

$$\max_{x_r, s_r, y_r} U^r(x_r, s_r, y_r)$$

$$\text{s.t. } N_r x_r + C_s N_r s_r + C_y y_r = N_r \omega_r$$

Denote the solution to this problem as

$$(x_r^{SS}, s_r^{SS}, y_r^{SS}).$$

In consequence, the reservation utility that the rich can insist on is the following:

$$\bar{U}^r \equiv U^r(x_r^{SS}, s_r^{SS}, y_r^{SS}). \tag{1}$$

Integrated City (IC)

Suppose now that the poor want the rich to live with them. By assumption, they have voting control of the city, and so their problem is to maximize the per-capita utility of the poor, while providing the rich exactly their reservation utility $\bar{U}_r$, and funding the public productions choices with a proportional wealth tax. Formally:

$$\max_{x_p, s_p, y_p, x_r, s_r} U^p(x_p, s_p, y_p)$$

$$\text{s.t. } N_p x_p + N_r x_r + C_s N_p s_p + C_s N_r s_r + C_y y_p = N_p \omega_p + N_r \omega_r \equiv W$$

$$\frac{x_r}{\omega_r} = \frac{x_p}{\omega_p}$$

$$U^r(x_r, s_r, y_p) = \bar{U}^r.$$

The first constraint is the material balance condition, expenditure equals total social wealth (W); the second constraint asserts that the fraction of the endowment which is consumed as private good is the same for rich and poor agents (and thus, implicitly, the tax is proportional); lastly, the third restriction is the participation constraint for the rich (where $\bar{U}^r$ comes from expression (1)). Substituting the tax constraint and rearranging terms, yields the following Lagrangian for problem (IC):

$$\mathcal{L} = U^p(x_p, s_p, y_p) + \lambda \left[W - \frac{W}{\omega_p} x_p - N_p C_s s_p - N_r C_s s_r - C_y y_p \right] +$$

$$\mu \left[\bar{U}^r - U^r(x_p \frac{\omega_r}{\omega_p}, s_r, y_p) \right]$$

The full set of first order necessary conditions for the problem is:

$$U^p_{x_p} - \lambda \frac{W}{\omega_p} - \mu U^r_{x_r} \frac{\omega_r}{\omega_p} = 0 \tag{2}$$

$$U^p_{s_p} - \lambda C_s N_p = 0 \tag{3}$$

$$U^p_{y_p} - \lambda C_y - \mu U^r_{y_p} = 0 \tag{4}$$

$$-\lambda C_s N_r - \mu U^r_{s_r} = 0 \tag{5}$$

$$N_p \omega_p + N_r \omega_r - \frac{W}{\omega_p} x_p - N_p C_s s_p - N_r C_s s_r - C_y y_p = 0$$

$$\bar{U}^r - U^r(x_p \frac{\omega_r}{\omega_p}, s_r, y_p) = 0$$

Using these first order conditions, it is straightforward to show that the equilibrium in the integrated city is not first best.

Lemma 1. *In general, neither public goods nor public services are efficiently provided in the integrated city.*

Proof/

For public service provided at the efficient level, the following conditions have to hold:

$$\frac{U^p_{s_p}}{U^p_{x_p}} = C_s$$

$$\frac{U^r_{s_r}}{U^r_{x_p}} = C_s$$

In other words, the marginal rate of substitution has to equal the marginal rate of transformation. It is straightforward to check that in our case:

$$\frac{U^p_{s_p}}{U^p_{x_p}} = \frac{C_s}{\frac{W}{N_p \omega_p} + \frac{\mu}{\lambda} \frac{1}{N_p} \frac{\omega_r}{\omega_p} U^r_{x_r}}.$$

Obviously, this will not be satisfied in general.

For public goods, the following must hold:

$$N_p \frac{U^p_{y_p}}{U^p_{x_p}} + N_r \frac{U^r_{y_p}}{U^r_{x_r}} = C_y$$

Looking at the first order conditions (2) and (4) of the integrated city problem, it is clear that this will also fail in general.

■

The next lemma shows that the reason for this inefficiency is the constraint that taxes must be proportional to income, and must not differentiate between rich and poor. A similar point is made in Stiglitz (1977)[8] who works with two groups of people, rich and poor, two types of goods (a private good and a pure public good) and uniform proportional taxation. He is mainly interested in the conditions where the two groups reside apart of each other ("exclusion points"), so that the poor do not "free ride" on the provision of the public good mainly financed by the rich. Our case adds public services, which provides a policy instrument that allows for differentiation between groups we want to provide differentially to each group. In contrast to Stiglitz, however, we are concerned with how this might be allow the rich and poor to live together in the socially efficient way rather than apart.

Lemma 2. *If the equal tax constraint is not present, the overall allocation is efficient.*

Proof/

If there is no tax constraint in the integrated city (which in our case is a constraint in the consumption of the private good x), then the amount of private good that the rich consume becomes an independent choice variable. Then, we rewrite condition (2) above as:

$$U^p_{x_p} - \lambda N_p = 0, \qquad (2')$$

[8] Section IV.5. This is also briefly described in Atkinson and Stiglitz (1980), Lecture 17, pp. 548-550.

condition (4) remains the same:

$$U^p_{y_p} - \lambda C_y - \mu U^r_{y_p} = 0, \tag{4}$$

and we add the condition for the optimal choice of the rich:

$$-\lambda N_r - \mu U^r_{x_r} = 0 \tag{8}$$

Solving for λ and μ in similar way as in the previous lemma, we obtain:

$$N_p \frac{U^p_{y_p}}{U^p_{x_p}} + N_r \frac{U^r_{y_p}}{U^r_{x_r}} = C_y$$

which is the condition for the private good and public good.

The Samuelson condition for the public services and public good can be obtained in exactly the same fashion. Furthermore, it is straightforward to check that the marginal rates of substitution of the rich and poor between the private good and the public services are the same, and equal their marginal rate of transformation.

Thus, without the tax constraint, the allocation is overall efficient.

■

Basically, what is going on in this model is that when the rich move to the city, they get to add their public goods contributions to those of the poor. The poor are then able to extract the rent gained from the per capita cost advantage that the larger city population has in public good provision. In effect, the rich can be made to pay their entire consumer surplus for this *increment* to public goods consumption. Based on this and the first order conditions the following observations regarding the public good levels in the integrated city are immediate.

Observation 1. If the poor get richer, the public goods level goes up in the integrated city. Given diminishing marginal utility, this leads to a greater willingness to pay for public goods on the part of the poor ($U^p_{y_p}$ goes up) and thus it is immediate that the first order conditions imply that more public good will be produced as a result.

Observation 2. If the rich get richer, the public goods level goes up in the integrated city. This is due to the fact that (a) at least under standard conditions of diminishing marginal utility this leads to a greater willingness to pay for public goods on the part of the rich ($U^r_{y_p}$ goes up), and (b) the participation constraint on the rich (μ) gets more binding (negative). Thus, the first order conditions imply that more public good must be provided.

Observation 3. If either the rich or the poor get more numerous, the public goods level goes up in the integrated city. This is for the same reasons as the two arguments given above.

The following observation regarding the welfare of agents in the integrated city is also immediate.

Observation 4. If either the rich or the poor get more numerous or more wealthy, their welfare goes up.

Unfortunately, there is not much more we can say with certainty in this general formulation of the model. Suppose, for example, the rich got richer. On one hand, the rich do better for themselves in the suburbs and thus can insist on a higher public services level when they move to the city. On the other hand, the fact that they are wealthier means they may (depending on the form of their utility function) consume more public and private goods in the integrated city. Thus, the marginal utility of both goes down. If the marginal utility of private good consumption goes down quickly, while the marginal utility of public good consumption goes down more slowly, the poor may be able to extract a high private good price for the extra public good the city provides. Thus, it is unclear if the poor's public services level goes up or down as the rich get richer. It is also unclear if the poor are better or worse off as the rich get richer. Suppose instead the number of rich people increases. A larger population enables them to do better in the suburbs, which in turn means a higher reservation utility. However, more rich means that there are more rich agents to enjoy the increment to public goods offered by the integrated city, which allows the poor to extract more rent. It is impossible to say which dominates in general. Similar arguments can be made for changes in the other parameters of the model.

4. Public Service Allocation and Welfare

Given that a general answer to many interesting questions is not available, we turn instead to a more specific example in hopes of making more progress. Assume now that all agents, both rich and poor, have the same Cobb-Douglas utility function,[9]

$$a \ln x + b \ln s + c \ln y.$$

Note that since this is a quasi-concave utility function and the technology is linear, it will not be necessary to check second order conditions. Given that all agents have the same Cobb-Douglas utility function, we are also guaranteed an interior solution. In general, this would not be the case. For example, it is conceivable for other utility functions that even if the rich received no private goods or public services, the public goods the poor were able to provide would still made the rich strictly better off than they would be in the segregated suburb. In this case, the participation constraint of the rich would not be binding.

4.1 Rich and Poor Alone

When the rich agents segregate themselves from the poor, to optimally allocate resources among themselves, they solve the following problem:

(SS)

$$\max_{x_r, s_r, y_r} \; a \ln x_r + b \ln s_r + c \ln y_r$$

$$\text{s.t.} \quad N_r \; x_r + C_s \; N_r \; s_r + C_y \; y_r = N_r \omega_r$$

This gives as a solution:

$$x_r^{SS} = \frac{a\omega_r}{a+b+c}, \; s_r^{SS} = \frac{b\omega_r}{C_s(a+b+c)}, \; y_r^{SS} = \frac{c \; N_r \; \omega_r}{C_y(a+b+c)}$$

[9] Faced with the choice of assuming a functional form for which closed form solutions are possible, or performing simulations with more sophisticated versions of the utility function, we chose the former. More comprehensive forms, like the CES utility function, might contribute to generality, but introduce non-linearities and we believe would not add much to the point we want to make in the paper.

Thus, the rich get the following utility when they live by themselves in a segregated suburb:

$$U^{SS} = \ln\left[\frac{a\omega_r}{a+b+c}\right]^a \left[\frac{b\omega_r}{C_s(a+b+c)}\right]^b \left[\frac{cN_r\omega_r}{C_y(a+b+c)}\right]^c$$

Of course, the poor will have to provide the rich with enough public services to meet this outside utility offer in the rich are to be persuaded to stay in the integrated city.

Similarly, the poor face the problem:

$$\textbf{(SC)} \qquad \max_{x_p, s_p, y_p} \; a\ln x_p + b\ln s_p + c\ln y_p$$

$$\text{s.t. } N_p \; x_p + C_s \; N_p s_p + C_y \; y_p = N_p \omega_p$$

This gives as a solution:

$$x_p^{SC} = \frac{a\omega_p}{a+b+c}, \; s_p^{SC} = \frac{b\omega_p}{C_s(a+b+c)}, \; y_p^{SC} = \frac{c \; N_p \; \omega_p}{C_y(a+b+c)}$$

Thus, the poor get the following utility when they live in a separate community:

$$U^{SC} = \ln\left[\frac{a\omega_p}{a+b+c}\right]^a \left[\frac{b\omega_p}{C_s(a+b+c)}\right]^b \left[\frac{cN_p\omega_p}{C_y(a+b+c)}\right]^c$$

4.2 Rich and Poor Together With a Differential Allocation of Public Services

Here we consider the case where both income groups live in one jurisdiction. Remember that the poor are the controlling majority of the city government. This is the reason why we assume that the city government will only consider the lower income group's welfare when making the decision on how to optimally allocate resources. The city government will, however, have the restriction to make the rich as well off as they are in the segregated suburbs. Given this, the city government's problem is:

$$\textbf{(IC)} \qquad \max_{x_p, s_p, y_p, x_r, s_r} \left[a\ln x_p + b\ln s_p + c\ln y_p\right] \; \text{s.t.}$$

$$N_r \; x_r + N_p \; x_p + C_s \; (N_r \; s_r + N_p \; s_p) + C_y \; y_p = N_r \; \omega_r + N_p \; \omega_p$$

$$\frac{x_p}{\omega_p} = \frac{x_r}{\omega_r}$$

$$a \ln x_r + b \ln s_r + c \ln y_p = \ln \left[\frac{a\omega_r}{a+b+c}\right]^a \left[\frac{b\omega_r}{C_s(a+b+c)}\right]^b \left[\frac{cN_r\omega_r}{C_y(a+b+c)}\right]^c$$

Define $\theta = \frac{W}{N_r\omega_r}$, which is the inverse of the share of the rich in total wealth. The solution for this problem is:

$$x_p^{IC} = \frac{a\omega_p}{a+b+c}$$

$$s_p^{IC} = \frac{bW\left(\theta^{\frac{b+c}{b}} - 1\right)}{C_s N_p (a+b+c)\theta^{\frac{b+c}{b}}}$$

$$y_p^{IC} = y_r^{IC} = \frac{cW}{C_y(a+b+c)}$$

$$s_r^{IC} = \frac{bW}{C_s N_r (a+b+c)\theta^{\frac{b+c}{b}}}$$

$$x_r^{IC} = \frac{a\omega_r}{a+b+c}$$

Lemma 1 showed that in general, public goods were not efficiently provided in equilibrium. Lemma 3, in contrast, shows that for the special case of Cobb-Douglas consumers, efficiency is regained.

Lemma 3. *Equilibrium provision of the public good in the integrated city satisfies the Samuelson condition.*

Proof/

Given that agents have Cobb-Douglas utility functions, it is immediate that marginal rate of substitution of private for public good either rich or poor agents is:

$$\frac{U_y}{U_x} = \frac{\frac{c}{y}}{\frac{a}{x}}.$$

Substituting the solution from the previous page and multiplying by the appropriate population gives the Samuelson condition. Simplifying we obtain:

$$C_y\left(\frac{N_p\omega_p}{W} + \frac{N_r\omega_r}{W}\right) = C_y$$

which is clearly true.

Following the same logic, the marginal rate of substitution of public services for public good for any agents is:

$$\frac{U_y}{U_s} = \frac{\frac{c}{y}}{\frac{b}{s}}.$$

Substituting the solutions obtained above for the rich and poor into the expression of the Samuelson condition, we obtain:

$$N_p \frac{c}{y_p}\frac{s_p}{b} + N_r \frac{c}{y_p}\frac{s_r}{b} = \frac{C_y}{C_s}\left[\frac{\theta^{\frac{b+c}{b}} - 1 + 1}{\theta^{\frac{b+c}{b}}}\right] = \frac{C_y}{C_s}$$

which is, again, true.

■

Note, however, just as in the general case, public services may not be the first best levels even with Cobb-Douglas preferences.

4.3 Public Services for Rich and Poor

The main question we wish to address in this paper is whether or not there are circumstances in which the poor would be willing to give a larger allocation of public services to the rich to attract them to their jurisdiction. Formally, we want to know if:

$$s_r^{IC} > s_p^{IC}$$

is true or not. Substituting in the solutions found above, the question becomes: is the following inequality

$$\frac{bW}{C_s N_r (a+b+c)\theta^{\frac{b+c}{b}}} > \frac{bW\left(\theta^{\frac{b+c}{b}} - 1\right)}{C_s N_p (a+b+c)\theta^{\frac{b+c}{b}}} \tag{10}$$

true? A little algebraic manipulation reduces this to the following expression.

$$\frac{N_r}{N_r + N_p} < \left[\frac{N_r \omega_r}{N_r \omega_r + N_p \omega_p}\right]^{\frac{b+c}{b}}. \tag{11}$$

Thus, inequality (10) holds if and only if inequality (11) holds.

Recall that by assumption, the rich are both wealthier and less numerous than the poor. As a benchmark, consider the boundary case where the rich and poor are equally wealthy, and equally numerous ($N_r = N_p$, and $\omega_r = \omega_p$). Then (11) becomes:

$$\frac{1}{2} < \left[\frac{1}{2}\right]^{\frac{b+c}{b}}.$$

Given that the exponent is greater than one, we see that this inequality is false. Thus, when the "rich" and "poor" are apparently similar in number and wealth, the rich get less public services. In other words, the poor are in a position to exploit the rich. All the poor have to do is make the rich as well off as they could be on their own. When the rich and the poor live together, the poor capture all the benefits of providing higher public goods production at lower per capita costs in the larger community. Recall that this is because the poor are assumed to have a voting majority. The only time this is not true is when the preference parameter for public goods is zero ($c = 0$). In this case (11) becomes an equality and the rich and poor are treated equally. Of course, this makes sense because the poor no longer can use their public goods production advantage as leverage over the rich. Note that since the inequality is continuous in the parameters, these arguments also hold when the rich are similar to the poor instead of identical.

Now consider what happens to (11) as the rich become relatively wealthier. The right hand side (RHS) converges to one, while the left hand side (LHS) is bounded above by one half. Thus, regardless of the size of the exponent, if the rich are wealthy enough, the inequality holds and the rich get more public services. Again, this makes sense. The richer the rich are, the better they can do on their own, *and* the more the poor want them. Thus, the rich are able to extract a better offer. It is also easy to see that as the desire for public goods increases (c goes to infinity) the RHS goes to zero, while the LHS in unchanged. Thus, for large values of c, the inequality does not hold. The intuition is that if the rich like public goods a great deal, they get large benefits from living with the poor in a larger community. This allows the poor to extract more rent, which they do in the form of lowering the public services level to the rich. Finally, suppose that the number of rich goes to zero. Inverting and simplifying, we can rewrite (11) as follows:

$$1 + \frac{N_p}{N_r} > \left[1 + \frac{N_p \omega_p}{N_r \omega_r}\right]^{\frac{b+c}{b}} .$$

Given that the exponent is greater than one, the fraction in the RHS converges to infinity at an exponential rate compared to the LHS. It follows that inequality (11) is false for sufficiently small N_r, and thus, the rich get less public services than the poor when they are a small fraction of the population. Again, this is intuitive. When the rich are small in number, they cannot produce much public good living alone in the suburbs, and this gives the poor a great deal of leverage.

We summarize these results in the following proposition.

Proposition 1. *(1) If the rich and poor are equal shares of the population, have the same wealth, and have no taste for public good, they get equal levels of public services in the integrated city. (2) If the rich are sufficiently wealthy compared to the poor or are sufficiently indifferent to public good, the rich receive higher levels of public services than the poor in the integrated city. (3) If the rich are a sufficiently small proportion of the population or have a sufficiently large taste for public good, the rich receive lower levels of public services than the poor in the integrated city.*

Proof/

Above.

■

The whole previous exercise would be meaningless if the poor did not clearly benefit from the presence of the rich. The next proposition shows that the poor will always be better off even after they have meet the utility constraint that is necessary to induce the rich to join them.

Proposition 2. *The utility of the poor is higher when living with the rich than forming a jurisdiction on their own.*

Proof/

Formally we have to show that $U_p^{IC} > U_p^{SC}$. Substituting in for the respective solutions this inequality becomes:

$$\ln\left[\frac{a\omega_p}{a+b+c}\right]^a \left[\frac{bW\left(\theta^{\frac{b+c}{b}}-1\right)}{C_s N_p(a+b+c)\theta^{\frac{b+c}{b}}}\right]^b \left[\frac{cW}{C_y(a+b+c)}\right]^c >$$

$$\ln\left[\frac{a\omega_p}{a+b+c}\right]^a \left[\frac{b\omega_p}{C_s(a+b+c)}\right]^b \left[\frac{cN_p\omega_p}{C_y(a+b+c)}\right]^c$$

Subtracting and combining terms gives us:

$$b\ln\left[\frac{bW\left(\theta^{\frac{b+c}{b}}-1\right)}{C_s N_p(a+b+c)\theta^{\frac{b+c}{b}}}\right]\left[\frac{C_s(a+b+c)}{b\omega_p}\right]+$$

$$c\ln\left[\frac{cW}{C_y(a+b+c)}\right]\left[\frac{C_y(a+b+c)}{cN_p\omega_p}\right] > 0,$$

which reduces to:

$$b\ln\left[\frac{W\left(\theta^{\frac{b+c}{b}}-1\right)}{N_p\omega_p\theta^{\frac{b+c}{b}}}\right] + c\ln\left[\frac{W}{N_p\omega_p}\right] > 0.$$

We can write this as:

$$(b+c)\ln\left[\frac{W}{N_p\omega_p}\right] + b\ln\left[\frac{\left(\theta^{\frac{b+c}{b}}-1\right)}{\theta^{\frac{b+c}{b}}}\right] > 0$$

Applying the exponential function we obtain:

$$\left[\frac{W}{N_p\omega_p}\right]^{b+c} \cdot \left[\frac{\theta^{\frac{b+c}{b}}-1}{\theta^{\frac{b+c}{b}}}\right]^b > 1$$

Taking b^{th}-root on both sides and using the fact that $\theta = \frac{W}{N_r\omega_r}$ we have that:

$$\left[\frac{W}{N_p\omega_p}\right]^{\frac{b+c}{b}} \cdot \left[\frac{W^{\frac{b+c}{b}} - (N_r\omega_r)^{\frac{b+c}{b}}}{W^{\frac{b+c}{b}}}\right] > 1.$$

This simplifies to:

$$\left[\frac{W}{N_p\omega_p}\right]^{\frac{b+c}{b}} - \left[\frac{N_r\omega_r}{N_p\omega_p}\right]^{\frac{b+c}{b}} > 1$$

or

$$\left[\frac{W}{N_p\omega_p}\right]^{\frac{b+c}{b}} > 1 + \left[\frac{N_r\omega_r}{N_p\omega_p}\right]^{\frac{b+c}{b}} .$$

Multiplying through by the denominator gives us:

$$W^{\frac{b+c}{b}} > (N_p\omega_p)^{\frac{b+c}{b}} + (N_r\omega_r)^{\frac{b+c}{b}} .$$

Since the exponent is bigger than one and the bases are positive, it is clear that this expression is true.

■

Finally, an interesting question relates to the level of public services that the rich and poor obtain while living in the same jurisdiction, compared to the level when both groups segregate themselves. Interestingly, for the higher income group, the level of public services is lower than if they go alone. For the poor, however, the level of public services is higher. We show this in the next two propositions.

Proposition 3. *The level of public services level of the rich in the integrated city is less than if they form their own jurisdiction. The level of public good, however, is higher.*

Proof/

We have to show that:

$$s_r^{IC} = \frac{bW}{C_s N_r(a+b+c)\theta^{\frac{b+c}{b}}} < s_r^{SC} = \frac{b\omega_r}{C_s(a+b+c)}$$

Simplifying these expressions we obtain:

$$\frac{W}{N_r\ \omega_r} < \left(\frac{W}{N_r\ \omega_r}\right)^{\frac{b+c}{b}}$$

This inequality is always true, since $\frac{W}{N_r\ \omega_r} > 1$ and $\frac{b+c}{b} > 1$.

On the other hand, for the case of the local public good, we have to show that:

$$y_r^{IC} = \frac{c\ W}{C_y(a+b+c)} > y_r^{SS} = \frac{c\ N_r\ \omega_r}{C_y(a+b+c)}$$

This is obvious, since $W > N_r \omega_r$.

■

A related question is whether the poor get more public services when they live with the rich or on their own. The positive answer is given by the following proposition:

Proposition 4. *The amount of public services the poor get living with the rich is higher than when forming a jurisdiction on their own.*

Proof/

We have to show that

$$s_p^{IC} > s_p^{SC}$$

Substituting in our results above:

$$\frac{bW\left(\theta^{\frac{b+c}{b}} - 1\right)}{C_s N_p (a+b+c)\theta^{\frac{b+c}{b}}} > \frac{b\omega_p}{C_s(a+b+c)}$$

Simplifying we obtain:

$$W - \frac{W}{\theta^{\frac{b+c}{b}}} > \omega_p N_p.$$

Recalling that $W = \omega_p N_p + \omega_r N_r$ and $\theta = \frac{W}{\omega_r N_r}$, this reduces to:

$$\theta^{\frac{b+c}{b}} > \theta.$$

Clearly, since $\theta > 1$ and $\frac{b+c}{b} > 1$, this inequality is true.

■

The reason for this result is simple. On their own, the rich consume a higher level of public services. In the case of the integrated city, the participation constraint of the rich has to be satisfied with equality. Since the city provides a higher level of public good than what the rich are able to produce when on their own, their consumption of public services decreases. Furthermore, resources of the integrated city (which, recall, is run by the poor) are enough to increase the allocation of public services to the poor.

5. Conclusion

The purpose of this paper has been to explain why it might be a political equilibrium for the rich to be treated better than the poor with respect to municipal services, despite the poor forming the voting majority of most cities. We show that the fact that the rich have the option of leaving the city for the suburbs gives them leverage, which allows them to extract resources from local governments whose interests are served by keeping wealthy taxpayers within the tax base. This leverage, however, cuts both ways. Cities, with their large populations, have a natural cost advantage in providing public goods. Since local governments only have to make the rich indifferent between the city and suburbs, the poor can extract from the rich the entire welfare differential between the smaller suburban public goods bundle and the larger one provided by the cities.

In the benchmark Cobb-Douglas case, when the rich are relatively numerous or wealthy, they end up receiving larger levels of public services than the poor. Conversely, when the rich are relatively few or not much wealthier than the poor, they may end up with a smaller public services allocation than the poor and yet still find that they are better off living with the poor in the city. The intuition is that the better the outside offer the rich are able to generate for themselves if they were to move to the suburbs, the better the treatment they get in the integrated city. We also show that the larger the taste for public good, the less public services the rich can expect to receive in the integrated city. This is quite simply because the value of a large population over which to share the cost of public goods becomes greater to the rich and this allows the city to extract larger rents for the poor.

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