

# Crime and Ethics

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January 2006

Abstract: We consider a simple model in which agents are endowed with heterogeneous abilities and differing degrees of honesty. Agents choose either to commit (property) crimes or invest in education and become workers instead. The model is closed in that all criminal proceeds are stolen from agents working in the formal sector and that expenditures on both deterrence and punishment of criminals are paid for through taxes levied on workers. Thus, although we assume that there are no direct interactive effects among criminals, criminals crowd each other in two ways: positively in that enforcement and punishment resources become more widely diffused as more agents commit crimes, and negatively in that the presence of more criminals implies that there is less loot to be divided over a larger number of thieves. We establish the possibility of multiple equilibria and characterize the equilibrium properties. We then evaluate the effectiveness of deterrence policies under a balanced government budget.

JEL Classification: I2, J2, J62, K42.

Keywords: Criminal Behavior, Educational Choice, Endogenous Sorting, Punishment.

Acknowledgment: We are grateful to comments and suggestions by Marcus Berliant, Derek Laing, Lance Lochner, Peter Rupert, Jay Wilson and Myrna Wooders. Needless to say, any remaining errors are solely the authors' responsibility.

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# 1 Introduction

In the United States, criminal activity has been geographically concentrated, associated with low education, high unemployment and poverty.<sup>1</sup> Crime rates rose in the U.S. during the 1980s but then fell during the 1990s.<sup>2</sup> In 1990, about 2% of the U.S. workforce was incarcerated and about 7% of the workforce was incarcerated, paroled or on probation. The median number of reported street robberies in Los Angeles equaled 4 per 1000 residents, but 10% of neighborhoods had crime rates four times greater than the median.<sup>3</sup> While many studies have investigated the factors that might influence an individual to choose crime as an occupation, we are only beginning to consider the forces that might produce such differing equilibrium crime rates across time and place. The main purpose of this paper is to contribute to our understanding of this issue, focusing particularly on property crimes.

The earliest literature on the economics of crime considers what might be termed the “external incentives” for agents to choose illegal activity over work in the legitimate sector (cf. Becker [1], Davis [4], and Ehrlich [5]). The effects of pecuniary and nonpecuniary punishments imposed on criminals on their decision making and the effectiveness of these public policies are the central concerns.

More recently, economists have begun to shift their attention to “internal motivations” for criminal behavior.<sup>4</sup> For example, Sah [15] points out that the more criminals there are, the more wide-spread must be enforcement resources. He formalizes this positive (to criminals) spillover and terms it the “interdiction effect”. Freeman, Grogger and Sontselie [6]

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<sup>1</sup>For empirical evidence relating education, unemployment and income to criminal activity, see Grogger [9], Gould, Mustard and Weinberg [8] and Witte and Tauchen [17], respectively.

<sup>2</sup>Grogger [9] attributes the rise in the crime rate in the 1980s to the drop in the real wage rate for the youth, whereas İmrohoroglu, Merlo and Rupert [12] regard the subsequent decline as a consequence of higher police enforcement.

<sup>3</sup>The geographical concentration of criminal activity has been documented by Freeman, Grogger and Sontselie [6] and Glaeser, Sacerdote and Scheinkman [7], among many others.

<sup>4</sup>The terminology of external incentives and internal motivations are taken from Rasmussen [14]. Internal motivations arise either from things that are internal to the agent (preferences or propensities, for example) or from interactions between agents. This is distinguished from external actions of governments that affect criminal behavior.

consider the “crowding-out” effect that the more criminals there are, the less loot for each criminal there is. Glaeser, Sacerdote and Scheinkman [7] model peer spillovers of criminal behavior, exploring how the presence of criminals can influence others to choose a life of a crime as well. İmrohorođlu, Merlo, and Rupert [11] develop a competitive equilibrium model of crime with elastic labor supply, and latter, İmrohorođlu, Merlo and Rupert [12] construct a political-economy model to study the effects of redistribution and policing on crime activity, both assuming exogenously given worker skills. Burdett, Lagos, and Wright [3] and Huang, Laing, and Wang [10] use a search-theoretic framework to model criminal decisions for one-dimensional heterogeneous agents and homogeneous agents, respectively. Lochner [13] constructs a simple two-period life-cycle model to examine how the labor-market conditions affect crime and educational choices but without allowing the feedback effect that criminal activity can influence the net value of formal employment or criminal proceeds. In an independent work, Verdier and Zenou [16] consider agents distinguished by their “color” (black or white) as well as by their personal aversion to crime. When peoples’ perception is that blacks have lower aversion to crimes, they are offered with lower wages and reside more distantly from jobs, thus creating an equilibrium in which poor blacks commit crimes. A very common finding in this literature is the presence of multiple equilibria with the coexistence of a high-crime equilibrium and a crime-free equilibrium.

One important factor that this literature seems to neglect is that agents may innately have different fundamental levels of honesty. Agents with weak ethics are naturally more likely to commit crime in all circumstances, although this will also interact with the abilities and other opportunities facing the agents. One could interpret the peer effects discussed in Glaeser *et al.* [7] as being related to this. Specifically, one might think of bad peers as weakening the ethics of the agents they interact with and causing them to follow their example. One may regard the measure of personal aversion to crimes in Verdier and Zenou [16] as honesty. Nonetheless, their measure of honesty is essentially tied to the public’s perceptions about people of different colors. It would be particularly interesting to investigate the story of interactive ethics formation in a multi-period model (the road to perdition?). Our approach here, however, is somewhat more modest. We consider only a static model in which agents arrive with a given level of honesty and explore how their choices are informed

by this internal moral compass. In contrast with Freeman *et al.*, Glaeser *et al.* and Verdier and Zenou, we consider two dimensions of heterogeneity in *both* ethics and abilities, and allow them to jointly affect the decisions by individuals as well as the nature of market equilibrium.<sup>5</sup>

The existing literature also seems to be incomplete in its consideration of the general equilibrium effects of crime rates. A high crime rate creates both positive *and* negative incentives for additional agents to choose criminal behavior. On the positive side are the “interdiction effects” identified by Sah [15]: the more criminals there are, the less likely any individual one of them will be caught given a fixed level of enforcement expenditure. This might even lead to a social collapse in which chances of getting caught are so low that everyone finds it optimal to choose crime as an occupation. On the other hand, the loot taken by thieves must be produced by the rest of the economy. Thus, a higher fraction of criminals implies that there are fewer workers and so less total wealth to be stolen, which in turn must also be distributed over a larger number of criminals. This negative spillover, which tends to push the economy back to stable low-crime equilibrium, has not been formally explored in the literature. Closing the model in this way, however, also exposes an additional, under-explored, effect that generates instability: more agents choosing crime implies that there are fewer workers to pay the taxes needed to fund enforcement and punishment efforts. Thus, for a fixed level of expenditure, each time an agent chooses to become a criminal, taxes must increase on the remaining workers. This in turn makes being a worker less attractive than being a criminal, all else equal.

The 1989 NBER Inner City Youth Survey (Boston, Chicago and Philadelphia) found that approximately 50% of subjects reported having several chances a day to make illegal income. In Boston in particular, 63% of subjects reported earning significantly more income from criminal than legal activities. Exact statistical data is very hard to find, but it seems likely that most full time workers earn little or no income from criminal activities. On the other hand, it is not easy for people with criminal records to hold well paying jobs. In

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<sup>5</sup>It should be pointed out that the Glaeser *et al.* (1996) paper could just as easily be interpreted as suggesting that having bad peers lowers the social penalty for bad behavior and so might have nothing to do with ethics at all.

addition, the lifestyle and habits of mind that lead a person to choose to commit crimes also make it difficult to maintain strong contact with the legal sector. Of course, it probably is the case that many criminals engage in at least some legal work activities. Other authors have emphasized this aspect of time allocation between working and committing a crime, see Block and Heineke [2], for example. In this paper, however, we will take the view that most agents tend to specialize in either criminal or legal activities and approximate this by modeling the occupational choice as being binary.

In the next section we develop a general equilibrium framework to study endogenous sorting between working in the formal labor market and committing a crime. Agents are endowed with heterogeneous abilities and different degrees of honesty. We allow agents to choose their own educational levels in response to market forces and do not rely on any direct peer externalities to drive our results. The local government authorities counter criminal activity with two complementary deterrence policies: policing and punishment. Thus, our paper contributes to the existing literature by:

- Allowing for two-dimensional heterogeneity in work ability and in ethical honesty. Mathematically, a two-dimensional characteristic space significantly adds to the difficulty of the analysis. However, adding this richness enables us to:
  - (i) explore the interplay between abilities and preferences,
  - (ii) obtain multiple *interior* equilibria,
  - (iii) uncover more interesting effects of governmental policies.
- Including both educational and occupational choice in a general equilibrium model of crime with police enforcement and punishment. This framework enables us to:
  - (i) examine *both* the external and internal margins of criminal behavior,
  - (ii) identify a variety of new and sometimes unexpected feedbacks that illustrate the complexity of making crime policy.

The main findings of the paper are summarized as follows. First, higher ability agents choose more education and get more income as a result regardless of their ethical level.

Second, the indifference boundary in the ability-honesty space between work and crime is downward sloping. Thus, the set of criminals in the two dimensional agent-characteristic space is comprehensive. Third, while an all-crime equilibrium can never exist, there is always a no-crime equilibrium with low proceeds or under severe punishments. Not only may this no-crime equilibrium coexist with an interior equilibrium associated with a positive crime rate, there may coexist multiple interior equilibria associated with different positive crime rates. The latter finding contrasts with most studies in the existing literature of crimes (e.g., Freeman, *et al.* [6], and Glaeser *et al.* [7]). In contrast with previous studies, multiple equilibria arise without any geographical externalities (cf. Freeman, *et al.* [6] and Verdier and Zenou [16]), direct interpersonal spillovers (cf. Sah [15] and Glaeser *et al.* [7]), or matching externalities (Burdett *et al.* [3] and Laing *et al.* [10]). Our paper thus adds further insights toward understanding the source of equilibrium multiplicity in crimes. Our result also provides a plausible explanation for the differing crime rates observed across time and place, particularly the geographical concentration of crimes and the phenomenon of urban ghettos. Finally, lower proceeds or greater punishments discourage criminal behavior, whereas higher minimum wage only reduce the incentive of the less able to commit a crime.

## 2 The Basic Model

We consider a model with a continuum of individuals, each of whom possesses two basic characteristics: *intellectual ability* ( $a$ ) and *ethical honesty* ( $h$ ). We assume that these traits are uncorrelated and follow a joint uniform distribution:  $G(a, h)$  over the compact support  $[0, 1] \times [0, 1]$ . We denote the set of agents in the economy by  $\mathcal{I}$  and will identify individual agents by their characteristics; thus,  $(a, h) \in \mathcal{I}$ .

Agents choose either to join the labor force or become criminals. If they choose to work, the wage they receive depends both on their basic intellectual ability and the level of educational attainment or on-the-job learning effort ( $e \in [0, 1]$ ). The set of these choices will be denoted  $e(a, h)$ . Education or learning is equally costly for all agents and this cost is given by:  $C(e)$ . We assume that  $C(e) \geq 0$ ,  $C'(e) > 0$ ,  $C''(e) > 0$ , and  $\lim_{e \rightarrow 1} C'(e) = \infty$ . It may be noted that one may reinterpret  $e$  as work effort, bearing in mind that those involved

in criminal activity would choose lower effort devoted to formal jobs.

If an agent of type  $(a, h)$  chooses to work, he receives a gross compensation  $W = W_0 + W_1ae$ , where  $W_0$  is the minimum wage and  $W_1 > 0$ . Note that the variable compensation depends on the product of ability and education, but is unaffected by the honesty of the agent.

The compensation for an agent who turns to a life of crime is more complicated. We assume that it depends on the following:

- the fraction of agents who commit crimes:  $\kappa \in [0, 1]$ ,
- the total wealth of the society:  $Y$ ,
- the part of that social wealth that the criminal class as a whole steal, referred to as the *loot*:  $L$ ,
- the probability of getting caught:  $\Pi$ ,
- the aggregate jail expenditure:  $J$ ,
- the agent's level of honesty (the more honest, the less an agent enjoys his ill-gotten proceeds):  $h$ .

We develop this more formally as follows. Suppose we are at a sorting equilibrium in which a set of agents,  $\mathcal{I}^c \subset \mathcal{I}$ , have decided to become criminals and we denote the educational choice of any given worker agent  $(a, h) \in \mathcal{I}^w \equiv \mathcal{I} \setminus \mathcal{I}^c$  by  $e(a, h)$ . Then the total national wealth is:

$$Y = \int_{(a,h) \in \mathcal{I}^w} [W_0 + W_1ae(a, h) - C(e(a, h))] dG(a, h). \quad (1)$$

The more criminals in the society, the higher the fraction of net social wealth is stolen from honest workers. Let  $S(\kappa)$  give this fraction. Assume  $S'(\kappa) > 0$ ,  $S''(\kappa) \leq 0$ ,  $\lim_{\kappa \rightarrow 0} S(\kappa) = 0$  and  $\lim_{\kappa \rightarrow 1} S(\kappa) < 1$ .

To deter crime, working agents pay a flat proportional wage tax rate of  $\tau$  and the resulting revenue is divided between expenditure on *police* ( $P$ ) and *jails* ( $J$ ). Note that feasibility

requires that

$$\tau Y = P + J. \quad (2)$$

The effectiveness of police in catching criminals depends both on the level of expenditure, and the number of criminals. This is captured by:

$$\Pi(\kappa, P),$$

which gives the probability for a criminal to be caught as a function of two factors,  $\kappa$  and  $P$ . We assume:  $\partial\Pi/\partial\kappa < 0$  (the *interdiction* effect),  $\partial\Pi/\partial P > 0$ ,  $\partial^2\Pi/\partial(\kappa)^2 > 0$ ,  $\partial^2\Pi/\partial(P)^2 < 0$  (*diminishing returns* to enforcement),  $\lim_{\kappa \rightarrow 0} \Pi(\kappa, P) = 1$  and  $\lim_{\kappa \rightarrow 1} \Pi(\kappa, P) > 0$ .

We model spending on jails as a mechanism to punish criminals. Such spending allows society to impose a cost on criminals if they are caught. Of course, the more criminals are caught by the police, the more thinly these punishment expenditures must be spread. Thus, the cost of punishment to a given criminal is given by the following function:

$$\lambda \left( \frac{J}{\Pi(\kappa, P)\kappa} \right), \quad (3)$$

where  $\lambda' > 0$ .

### 3 Occupational Choice

In addition to policy enforcement and conviction, two other factors affect the reward to criminal behavior. First, all else equal, the more criminals, the more widely the loot has to be divided. To keep matters simple, we will assume that the loot is divided equally across criminals. Second, the more honest an agent, the more he discounts gains from criminal activity. Putting this together we get the following equations for net compensation to an agent of type  $(a, h)$  from choosing to work and receiving a wage ( $w$ ) or being a criminal and sharing the loot ( $\ell$ ):

$$w(a, h; \kappa; \tau, P, J) = (1 - S(\kappa))(1 - \tau) \max_e [W_0 + W_1 a e - C(e)] \quad (4)$$

$$\ell(a, h; \kappa; \tau, P, J) = \frac{1 - h}{\kappa} S(\kappa)(1 - \tau)Y - \Pi(\kappa, P)\lambda \left( \frac{J}{\Pi(\kappa, P)\kappa} \right). \quad (5)$$

While the return to work is the portion of after-tax wage income that is not stolen by criminals, the return to committing a crime equals the individual share of the loot (the first term of the RHS of (5)) net of the expected cost of being caught (the second term).

An individual's occupational choice therefore depends on the comparison between these net compensations. One would choose to work in the formal sector if  $w(a, h) > \ell(a, h)$  and to commit crimes if  $w(a, h) < \ell(a, h)$ .

Denoting the measure of a set by  $\mu$ , a *feasible state* of the economy is  $(\tau, P, J, \mathcal{I}^c, e)$  where  $\tau Y = P + J$ ,  $\kappa = \mu(\mathcal{I}^c)$  and  $Y$  is consistent with this  $\kappa$ . Thus, the set of agents who are indifferent between work and crime is defined by the following equation:

$$w(a, h; \kappa; \tau, P, J) = \ell(a, h; \kappa; \tau, P, J). \quad (6)$$

Call this locus the *Best Response Occupational Choice Boundary (BROCB)*. More specifically, given a particular level of the crime rate  $\kappa$  and a set of policy parameters  $(\tau, P, J)$  the *BROCB* gives the cutoff level of honesty  $h$  as a function of  $a$  between crime and work being optimal choices for agents. Of course, it may be that for a given  $a$ , all agents should either commit crimes or all agents should work in the formal sector (meaning that equation (6) can never be satisfied for this  $a$ ). We will therefore need to know the boundaries on the upper and lower side where occupational choice becomes trivial in this way.<sup>6</sup> Formally, let  $a^{\max}$  be such that for all  $a \geq a^{\max}$  and for all  $h \in [0, 1]$ ,  $w(a, h; \kappa; \tau, P, J) \geq \ell(a, h; \kappa; \tau, P, J)$  if this exists and 1 otherwise. Similarly, let  $a^{\min}$  be such that for all  $a \leq a^{\min}$  and for all  $h \in [0, 1]$ ,  $w(a, h; \kappa; \tau, P, J) \leq \ell(a, h; \kappa; \tau, P, J)$  if this exists and 0 otherwise. Now, we can define the BROCB as follows:

$$BROCB(a; \kappa; \tau, P, J) = \begin{cases} 0 & a^{\max} < a \leq 1 \\ h \text{ s.t. (6) is met} & a^{\min} \leq a \leq a^{\max} \\ 1 & 0 \leq a < a^{\min} \end{cases} \quad (7)$$

The benchmark case is plotted in Figure 1A where  $a^{\max}$  and  $a^{\min}$  do not exist. An alternative case with both  $a^{\max}$  and  $a^{\min}$  existent is depicted in Figure 1B. For brevity, we do not display two other possible cases with either  $a^{\max}$  or  $a^{\min}$  existent.

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<sup>6</sup>We will show below that the *BROCB* is downward sloping, and so once the *BROCB* goes out of bounds above or below, it stays out of bounds.

Note that this is a best response boundary in the sense that all agents take the parameters  $(\kappa, \tau, P, J)$  as given. Thus, it may be that the crime rate  $\kappa$  is not consistent with the number of agents who choose crime as the best response. Similarly, we do not require at this point that the tax rate  $\tau$  is consistent with  $P$  and  $J$  in the sense of budget balance. We will define an equilibrium occupational choice boundary in the next section.

We begin by showing that higher ability agents choose more education and get more income as a result.

**Lemma 1.** (Education) *For a feasible state of the economy  $(\tau, P, J, \mathcal{I}^c, e)$ , the optimal level of education for an agent who chooses to be a worker is increasing in ability. In addition, higher ability agents choosing optimal educational levels earn more by working than would lower ability agents making optimal education choices.*

**Proof.** For an agent  $(a, h)$ , the optimal educational level maximizes:

$$(1 - S(\kappa))(1 - \tau) [W_0 + W_1 a e - C(e)].$$

Differentiating with respect to  $e$  yields,

$$W_1 a = \frac{\partial C(e)}{\partial e}, \tag{8}$$

which is independent of  $h$ . Recall that  $C(e) \geq 0$ ,  $C'(e) > 0$ , and  $C''(e) > 0$ . Thus, for any two agents  $(a, h)$  and  $(\bar{a}, \bar{h})$  such that  $a < \bar{a}$ , if we assume that both agents work, the agent with the higher ability chooses a higher educational level, i.e.,  $(\bar{a}, \bar{h}) > e(a, h)$ .

Moreover, since  $(\bar{a}, \bar{h})$  could have chosen the same educational level as  $(a, h)$ , but found a higher level to be optimal, it must also be that

$$W_0 + W_1 a e(\bar{a}, \bar{h}) - C(e(\bar{a}, \bar{h})) > W_0 + W_1 a e(a, h) - C(e(a, h)),$$

which proves the second part of the lemma. ■

This lemma allows us to show the next theorem which says that the *BROCB* is a downward sloping line. Let  $\gg$  denote dominance in each argument (and  $\ll$  conversely).

**Theorem 1.** (Occupational Choice Boundary) *For any state of the economy  $(\tau, P, J, \mathcal{I}^c, e)$ ,  $BROCB(a; \kappa; \tau, P, J)$  is decreasing in  $a$ . Moreover, for all  $(\bar{a}, \bar{h}) \gg (a, h)$ ,  $(\bar{a}, \bar{h}) \in I^w$  and for all  $(\bar{a}, \bar{h}) \ll (a, h)$ ,  $(\bar{a}, \bar{h}) \in I^c$ .*

**Proof.** Let  $h = BROCB(a; \kappa; \tau, P, J)$ , and take any  $(\bar{a}, \bar{h}) \gg (a, h)$ . By Lemma 1, agent  $(\bar{a}, \bar{h})$  gets more compensation from working than  $(a, h)$ . Now consider the compensation from choosing crime for each agent. The only thing that changes in the right-hand-side of equation (5) is that more honest agents discount the proceeds from the crime more heavily (since  $1 - \bar{h} < 1 - h$ ). We conclude that work is strictly more attractive and crime strictly less attractive to agent  $(\bar{a}, \bar{h})$  than  $(a, h)$ .

Showing the opposite holds for any agent  $(\bar{a}, \bar{h}) \ll (a, h)$  follows a completely parallel argument. ■

This implies that if an agent  $(a, h)$  is just indifferent between work and crime, all agents with higher abilities and greater honesty choose to work and all agents with lesser ability and honesty choose to commit a crime. Thus,  $\mathcal{I}^c$  is a comprehensive set,  $\mathcal{I}^w$  is an inversely comprehensive set, and the  $BROCB$  is a downward sloping line separates the two.

**Remark:** What happens if we eliminate heterogeneity in ethical honesty? One may illustrate this special case using Figure 1A. Specifically, we now fix ethical honesty at  $h = h_0$ . Thus, there exists a critical value of intellectual ability  $a_0$ : those with  $a > a_0$  will work in the formal sector whereas those with  $a < a_0$  will become criminals. Thus, the  $BROCB$  degenerates to a single point.

## 4 Equilibrium

A feasible state  $(\tau, P, J, \mathcal{I}^c, e)$  is an *Endogenous Sorting Equilibrium* (ESE) if

- (i) (educational choice) for all  $(a, h) \in \mathcal{I}^w \equiv \mathcal{I} \setminus \mathcal{I}^c$ ,  $e(a, h)$  is an optimal choice taking everything else as given, i.e.,  $e(a, h) \in \arg \max_e (1 - S(\kappa))(1 - \tau) [W_0 + W_1 a e - C(e)]$ ;
- (ii) (occupational choice) for all  $(a, h) \in \mathcal{I}^w$ ,  $w(a, h) \geq \ell(a, h)$  and for all  $(a, h) \in \mathcal{I}^c$ ,  $w(a, h) \leq \ell(a, h)$ ;
- (iii) (equilibrium sorting)  $\int_0^1 BROCB(a; \kappa; \tau, P, J) da = \kappa = \mu(\mathcal{I}^c)$ .

In specifying equilibrium sorting, the reader should be reminded that the marginal distribution of  $a$  is uniform. Also note that in stating this notion of equilibrium, we are taking

police and jail expenditures as being chosen exogenously, and since all ESE are assumed to be feasible, taxes are then implied by the equation  $\tau Y = P + J$ . Alternatively, one could endogenize this as a political equilibrium. In the interest of simplicity, we put this aside for now and make them policy variables determined by a social planner.

By definition, the crime rate is given by,

$$\kappa = K(\kappa) \equiv \int_0^1 BROCB(a; \kappa; \tau, P, J) da, \quad (9)$$

which constitutes a fixed-point mapping of  $\kappa$ . Upon substituting in the fixed-point value of  $\kappa$  for each  $a$  given  $(\tau, P, J)$ , the *Equilibrium Occupational Choice Boundary (OCB)* can then be derived:  $OCB(a; \tau, P, J)$ . Under the normality condition stated above,  $\frac{\partial K(\kappa)}{\partial \kappa} > 0$  and one may have multiple fixed points with those satisfying  $\frac{\partial K(\kappa)}{\partial \kappa} < 1$  being stable. Figures 2A and 2B, respectively, plot the cases of a single stable interior ESE (point  $E$ ) and two stable interior ESE's (points  $E_1$  and  $E_2$ ).

It is obvious that an all-crime equilibrium with  $\kappa = 1$  can never exist, because in that case there would be no proceeds for the criminal to take away (which can also be seen from the fixed point mapping). However, there is always a no-crime equilibrium with  $\kappa = 0$  given low proceeds or under severe punishments:

**Theorem 2.** (No-Crime Equilibrium) *For a feasible state of the economy  $(\tau, P, J, \mathcal{I}^c, e)$ , a no-crime endogenous sorting equilibrium with  $\kappa = 0$  emerges as long as committing a crime is not too profitable.*

By “not too profitable”, we mean that some combination of high punishment cost, high policing rates and low looting return rate ( $S(\kappa)$ ) make crime a relatively unattractive choice. The consequence is we fall below a critical crime rate such that so few criminal remain to diffuse policing and punishment resources that no one finds a life of crime to be a worthwhile choice. This result is shown by “guess and verify” by using equations (4), (5) and (9). Specifically, we show  $\kappa = 0$  satisfies (9) and that at this point  $w(a, h; \kappa; \tau, P, J) < \ell(a, h; \kappa; \tau, P, J)$ .

Furthermore, a nondegenerate equilibrium may arise:

**Theorem 3.** (Nondegenerate Equilibrium) *For a feasible state of the economy  $(\tau, P, J, \mathcal{I}^c, e)$ , a nondegenerate endogenous sorting equilibrium with  $\kappa \in (0, 1)$  exists when committing a crime is sufficiently profitable.*

From the discussion above the meaning of “sufficiently profitable” is obvious, and again the result is established by guess and verify.

It also turns out that a nondegenerate equilibrium may coexist with the no-crime equilibrium (Figures 2A) and that two or more nondegenerate equilibria with positive crime rates may coexist (Figures 2B). We will establish these results by way of numerical examples in Section 5. The possibility of multiple nondegenerate equilibria depends crucially on the two-dimensional heterogeneity we introduce in this model. Should one fix ethical honesty at  $h = h_0$ , there exists a unique critical value of intellectual ability  $a_0$ . When  $a_0 > 0$ , the only nondegenerate equilibrium is associated with a crime rate  $\kappa = a_0$  which coexists with the no-crime equilibrium ( $\kappa = 0$ ). Our result of multiple nondegenerate equilibria provides a plausible explanation for the differing crime rates observed across time and place.

We can use the framework developed so far to consider a number of policy questions. We conclude this section with several remarks in this spirit, noting that many of the findings below depend crucially on the two-dimensional heterogeneity and the interplay of occupational and educational choice.

- **Targeting educational subsidies:** First of all, it can never be Pareto improving to give educational subsidies to those who would choose to become workers in equilibrium. This is because agents pay for education with pretax income in this model, so they already equate the marginal benefit and marginal cost of education at an equilibrium. Providing a subsidy, therefore, induces them to obtain too much education. It also increases tax rates which make work less attractive and so may induce more agents to choose crime. On the other hand, national income must go down (since the marginal unit of education costs more than it produces) which implies that there is less loot and so crime is also less attractive. As a result, the net effect of these subsidies on the crime rate is unclear. Second, it may be Pareto improving to subsidize agents who would otherwise choose to become criminals in equilibrium. These agents do not internalize the benefits that reduced crime affords to existing workers and subsidizing education may induce them to become workers as well. The higher taxes required to pay for the subsidy, however, make work less attractive, all else equal, and so may

induce existing workers to choose crime instead. The overall effect on crime rate is therefore also ambiguous. We nevertheless do learn something about the best place to target these subsidies if a society for whatever reason has decided to have them: they should be directed at agents who are high-ability (and so would use their education most productively) but who are dishonest and so might choose a life of crime if not given additional incentives. This suggests we should neither help the intellectually disadvantaged nor give merit-based scholarships. Instead we should use subsidies to encourage smart people with juvenile records to go to school (possibly via penal reform). We certainly should not take away scholarships from students who have drug or other convictions since these are exactly the agents who are on the edge who might end up being a criminal burden to society if they do not have extra incentives to stay in school.

- **Implications for balanced and unbalanced growth and contractions:** Another implication relates to unbalanced growth or contraction in an economy. If an economy grows in a balanced way with returns to both high and low-skill workers keeping in the same proportion, there is nothing to induce a move away from the current equilibrium crime rate. Crime and the formal sector remain equally attractive on the margin and so growth *per se* neither induces nor prevents crime. Symmetrically, recession and depression should not in themselves cause a social breakdown. On the other hand, if growth or contraction increases the rewards paid to high-skill workers or decreases those paid to low-skill ones disproportionately, it becomes relatively more attractive for low-ability workers to choose crime. This will increase the crime rate and may even cause the society to transit from a low crime to a high crime equilibrium. Thus, unbalanced growth can be seen as corrosive to social cohesion in the context of this general equilibrium crime model. As an aside, if during an economic contraction, earning opportunities decline but there remains accumulated wealth from better days that might be stolen, contractions would tend to push the society toward an equilibrium with higher crime rates.
- **Enforcement and education choice in a dynamic context:** So far, we have

neglected dynamic considerations. Suppose we extend this model to consider agents who live many periods but choose a life path early on. This may introduce additional instabilities to the model. For example, in choosing educational levels, agents need to project what they think the likely rewards to being a worker over the course of their lives are likely to be. If all agents hold optimistic priors about the future, high educational levels are chosen, national income is high, and crime rates and taxes are low. Once an agent has chosen a low educational level, however, honest labor becomes permanently less attractive. It may also be difficult to join the labor force once one has committed crimes. Thus, if the government has an optimistic prediction about future growth, it may be in its interests to subsidize education to keep pessimistic agents from closing off their futures as workers. It might also have more interest in vigorously enforcing laws against young agents to discourage irreversible criminal behavior and to be less concerned about the actions of older workers who would lose only a few productive years if they turned to crime. This might be a justification for aggressive enforcement of laws against violent and drug crimes and relatively mild punishments for white collar crimes.

## 5 Characterization

Since the ESE in the economy with two-dimensional heterogeneity is very difficult to characterize analytically, we will turn instead to a numerical approach. To be more concrete, let the education cost function be constant-elastic and the punishment cost function be linear:

$$C(e) = C_0 e^{1+\alpha} \quad \text{and} \quad \lambda \left( \frac{J}{\Pi\kappa} \right) = \lambda_0 \frac{J}{\Pi\kappa},$$

where  $\alpha > 0$ ,  $C_0 > 0$  and  $\lambda_0 > 0$ . We then write the educational choice function according to (8) as:

$$e = \varepsilon(a) \equiv \frac{W_1 a}{(1 + \alpha)C_0}, \tag{10}$$

and the pre-tax, pre-crime net earned income as:

$$W_0 + W_1 a e - C(e) = W_0 + B a^2 \tag{11}$$

where  $B \equiv \frac{1}{C_0} \left( \frac{W_1}{1+\alpha} \right)^2$ . Using (7) and (10), we can rewrite (1) as:

$$Y(\kappa) = \int_0^1 (1 - BROCB(a; \kappa; \tau, P, J)) [W_0 + W_1 a \varepsilon(a) - C(\varepsilon(a))] da, \quad (12)$$

where from (6) and (7),

$$\begin{aligned} & BROCB(a; \kappa; \tau, P, J) \\ &= \min \left\{ 1, \max \left\{ 0, 1 - \frac{\lambda_0 J + (1 - \tau)(1 - S(\kappa))\kappa [W_0 + W_1 a \varepsilon(a) - C(\varepsilon(a))]}{(1 - \tau)S(\kappa)Y(\kappa)} \right\} \right\}. \end{aligned} \quad (13)$$

It is clearly seen that  $sign \left[ \frac{\partial Y(\kappa)}{\partial \kappa} \right] = -sign \left[ \frac{\partial BROCB(a; \kappa; \tau, P, J)}{\partial \kappa} \right]$ . Substituting (11) and (13) into (12) and manipulating, we get:

$$\begin{aligned} & Y(\kappa) \\ &= \left( \frac{1}{S(\kappa)} \int_0^1 \left\{ \frac{\lambda_0 J}{1 - \tau} + (1 - S(\kappa))\kappa [W_0 + W_1 a e - C(e)] \right\} [W_0 + a e - C(e)] da \right)^{1/2} \\ &= \left[ \Upsilon_1 \frac{1}{S(\kappa)} + \Upsilon_2 (1 - S(\kappa))\kappa \right]^{1/2} \end{aligned} \quad (14)$$

where

$$\begin{aligned} \Upsilon_1 &\equiv \frac{\lambda_0 J}{(1 - \tau)} \left[ W_0 + \frac{1}{3C_0} \left( \frac{W_1}{2} \right)^2 \right] \\ \Upsilon_2 &\equiv (W_0)^2 + \frac{2W_0}{3C_0} \left( \frac{W_1}{2} \right)^2 + \frac{1}{5} \left( \frac{1}{C_0} \right)^2 \left( \frac{W_1}{2} \right)^4 \end{aligned}$$

Using (11) and (14), we can rewrite (13) to obtain:

$$\begin{aligned} & BROCB(a; \kappa; \tau, P, J) \\ &= \min \left\{ 1, \max \left\{ 0, 1 - \frac{\lambda_0 J + (1 - \tau)(1 - S(\kappa))\kappa (W_0 + Ba^2)}{(1 - \tau)S(\kappa)^{1/2} [\Upsilon_1 + \Upsilon_2 (1 - S(\kappa))\kappa S(\kappa)]^{1/2}} \right\} \right\} \end{aligned} \quad (15)$$

One can easily show  $\frac{\partial BROCB(a; \kappa; \tau, P, J)}{\partial a} < 0$  (i.e., downward sloping  $BROCB$ ). Yet, the sign of  $\frac{\partial BROCB(a; \kappa; \tau, P, J)}{\partial \kappa}$  remains ambiguous. If the functional form of the fraction of social wealth stolen from workers is so chosen to satisfy  $\frac{\partial [(1 - S(\kappa))\kappa]}{\partial \kappa} < 0 < \frac{\partial [(1 - S(\kappa))\kappa S(\kappa)]}{\partial \kappa}$ , however, one can establish:  $\frac{\partial Y(\kappa)}{\partial \kappa} < 0$  and  $\frac{\partial BROCB(a; \kappa; \tau, P, J)}{\partial \kappa} > 0$  for the interior range  $BROCB(a; \kappa; \tau, P, J) \in (0, 1)$ . Intuitively, this is a case where a “normality condition” is imposed so that crime is harmful for the society’s aggregate income. In this case, there exists

a  $\kappa_{\min}$  such that  $BROCB(a; \kappa; \tau, P, J) = 0$  for all  $\kappa < \kappa_{\min}$ . Moreover, even as  $\kappa \rightarrow 1$ , we have:  $BROCB(a; \kappa; \tau, P, J) = 1 - \frac{\lambda_0 J + (1-\tau)(1-S(1))(W_0 + Ba^2)}{(1-\tau)S(1)^{1/2}[\Upsilon_1 + \Upsilon_2(1-S(1))S(1)]^{1/2}} < 1$ .

We now specify further the arrest probability as

$$\Pi(\kappa, P) = 1 - \frac{\kappa^\delta}{\beta + \gamma P},$$

where  $\beta > 0$ ,  $\gamma > 0$  and  $\delta \in (0, 1)$ . We then specify the fraction of social wealth stolen from honest workers as

$$S(\kappa) = 1 - (1 + \sigma)^{-\kappa},$$

where  $\sigma > 0$ .

We next set in the benchmark case:  $\alpha = 1$ ,  $C_0 = 1$ ,  $\beta = 1$ ,  $\gamma = 0.5$ ,  $\sigma = 2.5$ ,  $\lambda_0 = 0.01$ ,  $\delta = 0.5$ ,  $W_0 = 10$ ,  $W_1 = 2$ ,  $P = 1.46$ , and  $J = 0.5$  (which imply a budget-balancing tax rate at  $\tau = 0.2$ ). Under these parameter values, a no-crime equilibrium always exists. Moreover, the OCB is relatively flat, as given in Figure 3, where all agents with  $h < 0.118$  become criminals and those with  $h > .159$  work in the formal sector. Thus, the benchmark interior crime rate is 14.5% and the coexistence of a no-crime equilibrium and a nondegenerate equilibrium is verified.

By performing comparative statics around the nondegenerate equilibrium (see Figure 3), we can establish an array of results which are robust to plausible variations in the parameters.

- **What happens if there is an exogenous increase in the cost of education  $C_0$ , an increase in the fixed wage  $W_0$ , or a reduction in the variable wage  $W_1$ ?** From (6), it is clear that all agents who work choose lower education levels and as a consequence, aggregate income decreases. This latter effect implies crime is less attractive, so the crime rate ( $\kappa$ ) may go up or down. We can illustrate this more fully using the  $BROCB$  given by (15): an increase in  $C_0$ , an increase in  $W_0$ , or a reduction in  $W_1$  lowers (raises) the interior range of the  $BROCB$  when  $a$  is small (large), thus causing a counter-clockwise rotation in the  $OCB$  (becoming flatter as plotted in Figure 3). This implies that a less able agent is less likely to commit a crime but a more able one is more likely to do so. Such *asymmetric* responses cannot

arise with one-dimensional heterogeneity. To be more concrete, by increasing  $W_0$  from 10 to 15, less able agents ( $a < 0.54$ ) are less likely to become criminals whereas more able agents ( $a > 0.54$ ) are more likely to withdraw from the formal sector.

- **What happens if rewards to crime ( $\sigma$ ) or punishment cost ( $\lambda_0$ ) changes?**

We find that the crime rate is most responsive to changes in  $\sigma$ , which measures the relative size of the proceeds: nothing (all) can be stolen as  $\sigma \rightarrow 0$  ( $\sigma \rightarrow \infty$ ). When we increase  $\sigma$  from 2.5 to 3 and 4, respectively, the society's crime rate rises to 21.0 and 30.5 percent (the *OCB* moves outward in Figure 3); as  $\sigma$  goes down to 2, the crime rate drops to 5.5%. If we further reduce  $\sigma$  to 1.9, the no-crime equilibrium emerges as the only equilibrium outcome. On the other hand, higher values of  $\lambda_0$  will shift the *OCB* inward (see Figure 3) and reduce the crime rate, because an increase in the punishment cost facing criminals discourages criminal activity. Raising  $\lambda_0$  from 0.01 to 0.15 is sufficient to remove anyone's incentive to commit a crime.

We turn next to identifying a set of parameters that may generate two nondegenerate equilibria. One straightforward case is associated with all the benchmark parameter values but with a much greater policing spending  $P = 8$ . In this case, two nondegenerate equilibria coexist where the high-crime equilibrium features  $\kappa^H = 10.2\%$  and the low-crime equilibrium features  $\kappa^L = 1.7\%$ . Our result of multiple interior equilibria provides a plausible explanation for the differing crime rates observed across time and place, particularly the geographical concentration of crimes and the phenomenon of urban ghettos

**Remark:** In the case where heterogeneity in ethical honesty is eliminated, the *BROCB* degenerates to a single point. Although there still coexist a nondegenerate and a no-crime equilibria, the coexistence of two nondegenerate equilibria can never arise. The *interplay of the two-dimensional heterogeneity* together with *endogenous educational and occupational choice* gives birth to multiple nondegenerate equilibria.

- **Effects of crime deterrence policies:** Consider an exercise where the expenditure on policing  $P$  or the expenditure on jails  $J$  increases. While chances of getting caught or punished for crime are higher, taxes must also go up to maintain government

balanced budget. Thus, crime is unambiguously less attractive, but work may be more or less attractive. So again the crime rate may go up or down, though locally around the computed nondegenerate equilibrium with  $(\tau, P, J) = (0.2, 1.46, 0.5)$ , the deterrence effect is always present (i.e.,  $\kappa$  reduces).

One might wonder if we can use the model to shed light on which deterrence policy is more effective. Moreover, how should the optimal policing level be chosen given that it has competing effects on the crime rates?

- **Crime deterrence policies compared:** Consider a budget-balancing shift in crime policy from  $(\tau, P, J) = (0.2, 1.46, 0.5)$  to  $(\tau, P, J) = (0.2, 0.96, 1)$ , that is, from more police enforcement to jail punishment. The overall crime rate changes only slightly, decreasing from 14.5% to 14.2%, and the *OCB* shifts inward uniformly. This suggests that jails are marginally more effective than policing. This result is quite robust locally around the computed nondegenerate equilibrium, due mainly to the probabilistic uncertainty and the interdiction effect.
- **On optimal policing spending:** Let us now set jail spending at the benchmark value  $J = 0.5$  and vary policing spending  $P$  from its benchmark value 1.46 (by adjusting the tax rate to balance government budget). As  $P$  increases (decreases), the *OCB* shifts outward (inward) uniformly, so the overall crime rate reduces (rises), but such changes are relatively small (ranging from 10 to 16%). However, once  $P$  increases above 7.6, two nondegenerate equilibria coexist and it is possible that the overall crime rate can be reduced dramatically to below 2%. Thus, as a result of multiple nondegenerate equilibria, changing policing spending sometimes causes a discrete jump from the high crime to the low crime equilibrium.

Two comments are now in order. First, we have attempted to conduct policy analysis by looking at different functional forms while choosing parameter values to reflect empirically realistic crime rates. What we discovered is that the results are highly dependant on the functional forms we chose, especially with regard to the punishment cost function,  $\lambda(\frac{J}{\Pi\kappa})$ . Second, due to endogenous occupational and educational choice and requirement of

a balanced government budget, a crime-rate minimizing policing policy need not be welfare maximizing.

A natural question to ask is how one might set optimal anti-crime policy in this context. One difficulty is that it is not clear how to weigh the welfare of workers and criminals. We experimented by considered equal weight, 50% discounting and zero weight on criminals. Not surprisingly, the more we weighted the welfare of the criminals, the more one favors a lower punishment level.

Even if we could resolve the welfare weight problem, a deeper and probably insurmountable problem remains. Recall that our model often produces multiple stable ESE. As a result, changing policy parameters can induce a discrete jump from high-crime to low-crime nondegenerate equilibrium. Of course, once at the low-crime equilibrium, a reduction in police spending may now reduce the crime rate (by encouraging work and education) and improve welfare. Yet to sustain such a path toward the no-crime equilibrium requires “self-fulfilling prophecies” in the sense that individuals adjust their expectations to believe that society is heading to a no-crime, high welfare state. Thus, there do not really exist welfare maximizing policies in a static sense. We can find locally optimal policies, and perhaps even welfare rank the stable equilibrium. Thinking about socially optimal policies, however, requires a dynamic structure and full specification of belief formation which is outside the scope of the current model.

**Remark:** For example, we have tried a nonlinear punishment cost function,  $\lambda(\frac{J}{\Pi\kappa}) = \lambda_0(\frac{J}{\Pi\kappa})^{1+\eta}$ , where  $\eta \geq 0$  (when  $\eta = 0$ , it reduces to the benchmark setup). In the case with  $\eta = 0.5$ , we can maintain all the other benchmark parameter values with  $(\tau, P, J) = (0.2, 1.46, 0.5)$  but set  $\beta = 0.25$  and  $\lambda_0 = 0,0036$  to obtain the benchmark (high-crime) equilibrium with  $\kappa = 14.5\%$ . However, a low-crime nondegenerate equilibrium with  $\kappa = 1.07\%$  now coexist – indeed, the multiplicity of nondegenerate equilibria arise for all reasonable levels of policing spending  $P \in [0.5, 10]$ , in contrast to the case of a linear punishment cost function. Additionally, there is an interior level of police spending,  $P = 4.01$ , achieving the lowest crime rate at  $\kappa = 0.71\%$ . Yet, for the arguments mentioned above, this policing policy need not be optimal in the welfare sense, as one may now further reduce the spending to improve welfare as long as individuals believe that the economy will not shift back to the

high-crime state.

In summary, an important implication of our analysis points to a recommendation for an optimal *dynamic* policy of heavy enforcement at the early stage to bring the society to a low spending, low crime, high welfare state.

## 6 Concluding Remarks

We considered an economy with a continuum of agents who have heterogeneous abilities and ethics. Agents must choose between acquiring education and becoming workers or forgoing education and becoming criminals. The model is closed in the sense that all loot stolen by the criminals must be produced by the workers who must also pay for any enforcement and punishment efforts through an income tax. As a result there are both positive and negative spillovers between criminals: positive as criminals draw police attention from one another, and negative as criminals must divide their fraction of the national product among all agents who share their occupational choice.

We show that high and low crime equilibria can exist for the same set of parameters. We also show that the indifference boundary in the ability-honesty characteristic space is downward sloping. Thus, high ability people find the formal sector more attractive than low ability people of the same ethical level. This implies, for example, that the average accountant is likely to be less fundamentally honest than the average convenience store clerk.

The model allows us to consider a number of policy questions. It suggests:

- That scholarships given on basis of merit may be socially wasteful since high ability agents choose to work and make optimal education choices already.
- Scholarships that induce agents who would have chosen a life of crime to go to school instead can be socially beneficial.
- Growth or contractions that affect all members of a society equally have no implication for equilibrium crime rates.

- Unbalanced growth or contraction that help the rich or hurt the poor disproportionately may lead to social breakdown.

There are many directions in which this work might be extended. For example, consider an overlapping generations version of this model in which the ethical level of agents is influenced by their peers and education choices made in early life affect work opportunities in later life. Both of these effects lead to instabilities and a tendency for agents with even small differences in their abilities and backgrounds to strongly diverge in their life choices. The vicious cycle of children raised in crime-ridden neighborhoods being more crime-prone and thus making irreversible decisions to drop out of school and then subsequently finding crime more attractive than work in later life may therefore emerge. (Children with the same characteristics but in better neighborhoods might be on the other side of the OCB and have radically different life paths). This would have strong implications for social policy (such as busing inner-city kids to wealthy suburbs and imposing very aggressive law enforcement in poor neighborhoods. Similarly, policies to manage unbalanced growth in order to prevent social breakdown and to bring countries in transition or which have experienced social breakdown back to order and growth could be explored in a dynamic context. The closing of the model, which introduces new and interesting feedbacks, and the inclusion of ethics or values, especially as they emerge dynamically, opens new and more realistic avenues to refine our understanding of these issues.

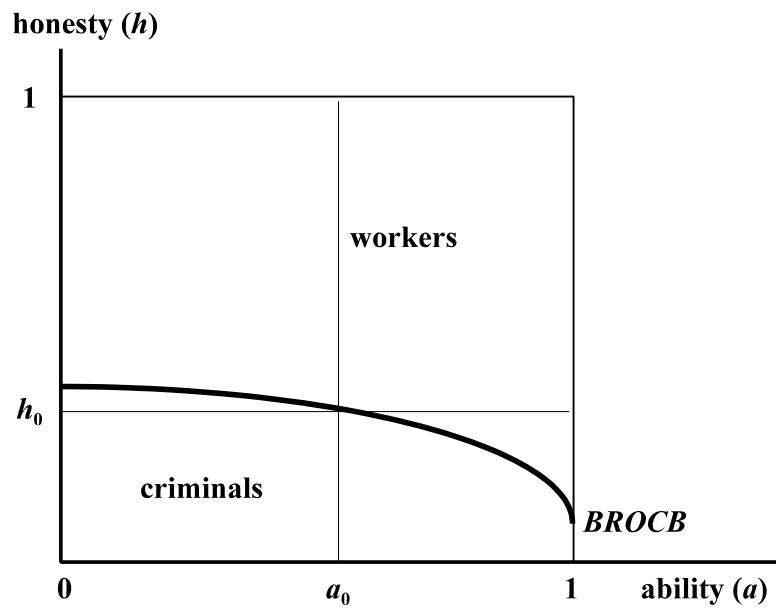
## References

- [1] G. S. Becker, Crime and punishment: an economic approach, *Journal of Political Economy* 76 (1968) 169-217.
- [2] M. K. Block, J. M. Heineke, A labor theoretic analysis of criminal choice, *American Economic Review* 65 (1975) 314-325.
- [3] K. Burdett, R. Lagos, R. Wright, Crime, inequality, and unemployment, *American Economic Review*, 93 (2003) 764-777.
- [4] M. L. Davis, Time and punishment: an intertemporal model of crime, *Journal of Political Economy*, 96 (1988) 383-390.
- [5] I. Ehrlich, Participation in illegitimate activities: a theoretical and empirical investigation, *Journal of Political Economy*, 83 (1973) 521-565.
- [6] S. Freeman, J. Grogger, J. Sonstelie, The spatial concentration of crime, *Journal of Urban Economics*, 40 (1996) 216-231.
- [7] E. L. Glaeser, B. Sacerdote, J. A. Scheinkman, Crime and social interactions, *Quarterly Journal of Economics*, 111 (1996) 507-548.
- [8] E. Gould, D. Mustard, B. Weinberg, Crime rates and local labor market opportunities in the United States, 1979-97, *Review of Economics and Statistics*, 84 (2002) 45-61.
- [9] J. Grogger, Market wages and youth crime, *Journal of Labor Economics*, 16 (1998) 756-791.
- [10] C. Huang, D. Laing, P. Wang, Crime and poverty: a search-theoretic approach, *International Economic Review*, 45 (2004) 909-938.
- [11] A. İmrohoroğlu, A. Merlo, P. Rupert, On the political economy of income redistribution and crime, *International Economic Review*, 41 (2000) 1-25.
- [12] A. İmrohoroğlu, A. Merlo, P. Rupert, What accounts for the decline in crime?, *International Economic Review*, 45 (2004) 707-729.

- [13] L. Lochner, Education, work and crime: a human capital approach, *International Economic Review*, 45 (1999) 811-843.
- [14] E. Rasmusen, Stigma and self-fulfilling expectations of criminality, *Journal of Law and Economics*, 24 (1996) 519-543.
- [15] R. K. Sah, Social osmosis and patterns of crime, *Journal of Political Economy*, 94 (1991) 1272-1295.
- [16] T. Verdier, Y. Zenou, Racial beliefs, location, and the causes of crime, *International Economic Review*, 45 (2004) 731-760.
- [17] A. D. Witte, H. Tauchen, Work and crime: an exploration using panel data, *Public Finance*, 49 (1994) 155-167.

Figure 1. Best Response Occupational Choice Boundary (*BROCB*)

A. The Benchmark Case



B. An Alternative Case

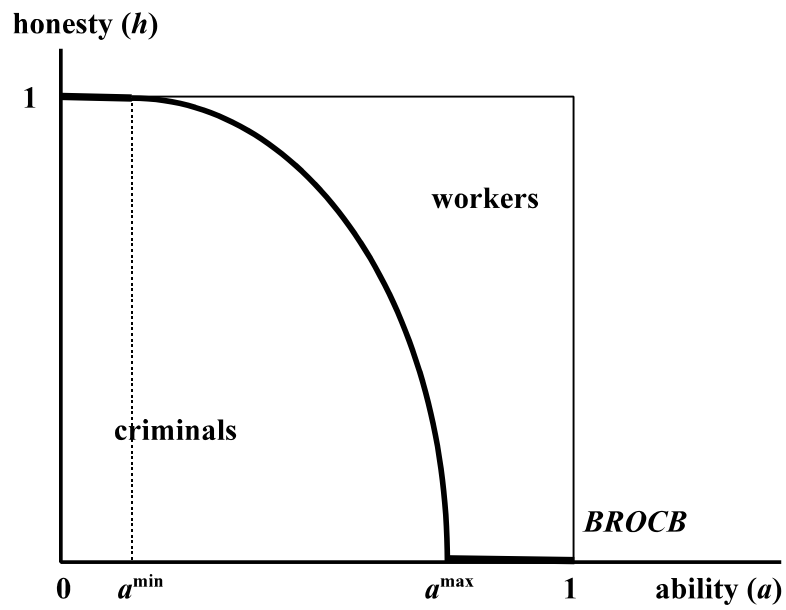
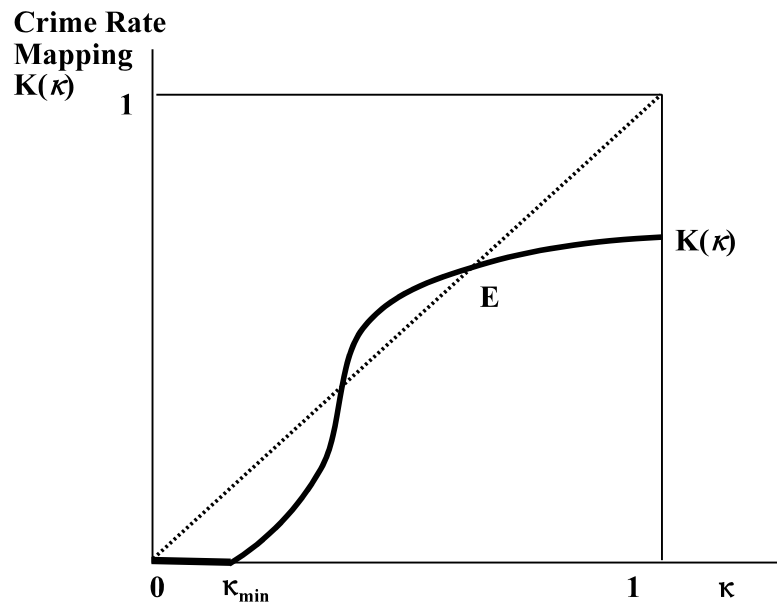
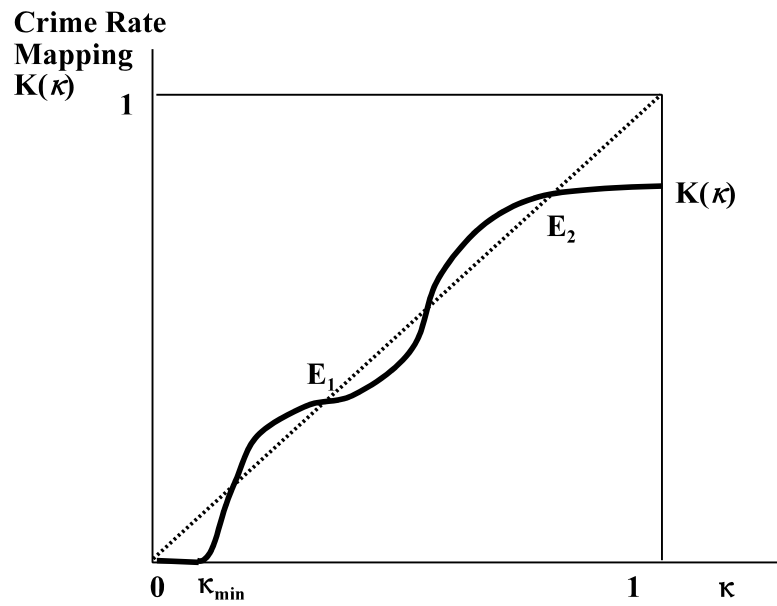


Figure 2. Equilibrium Crime Rate

A. Unique Interior Equilibrium



B. Multiple Interior Equilibria



**Figure 3. Changes in Equilibrium Occupational Choice Boundary (*OCB*) in Response to Parameter Shifts**

