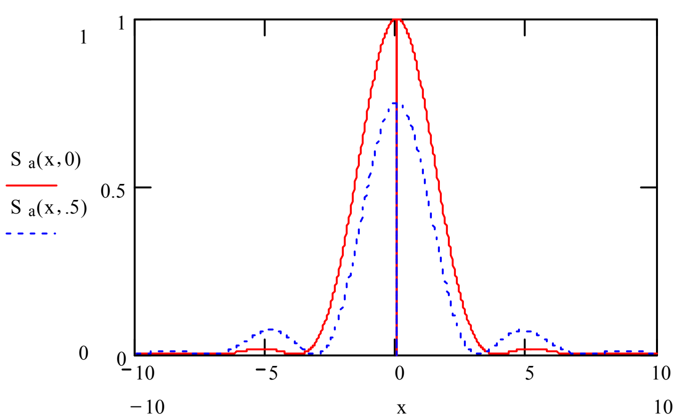
Exercise: Annular beams are frequently encountered in optical systems. For example, in Cassegranian telescope designs the center of the primary mirror is obscured by the secondary mirror, which reflects the beam through a hole in the center of the primary, and in edge-coupled, unstable optical resonators the output laser beam has an annular profile. The central obscuration diminishes the intensity of the beam on axis, even beyond the reduction in the total power in the beam, and introduces rings in the diffracted beam outside the central spot.

(a) Show that the amplitude of the beam in the far field is

$$a(\mathbf{r}) = i a_0 (\mathbf{k} \cdot \hat{\mathbf{n}}') \frac{e^{ikr}}{r} \left[ R_2^2 \frac{J_1(\Delta k R_2)}{\Delta k R_2} - R_1^2 \frac{J_1(\Delta k R_1)}{\Delta k R_1} \right]$$

where $R_1$ and $R_2$ are the inner and outer radii of the annulus.

(b) Using Mathematica, MathCad, or your favorite program, compute and plot the intensity pattern (corrected to normalize out the reduced area of the beam) for $R_1 = \frac{1}{2} R_2$, and compare your answer to the figure below.




(c) A convenient measure of the focusability of a laser beam is the Strehl ratio, defined as the ratio of the peak intensity of the focused beam to that of an ideal beam (in this case the unobscured beam). Show that after correcting for the reduced area, the Strehl ratio is

$$S = 1 - \frac{R_1^2}{R_2^2}$$