Section 10.1.5 Angular Distribution of Radiation from a Point Charge

Exercise: Consider a collision between an electron with charge q moving with velocity $\mathbf{v} = \beta c\hat{\mathbf{z}}$ parallel to the z axis through the point $x = b$ and a massive (stationary) charge Q located at the axis, as shown in the figure below. If the impact parameter b is large enough, the trajectory of the incident particle is nearly a straight line, but we can compute the instantaneous acceleration at each point on the trajectory from the electric field of the stationary charge at that point, and from this compute the power being radiated.
(a) Using the nonrelativistic formula for $d\mathcal{P}/d\Omega$, show that the total energy (Bremsstrahlung) radiated in the direction $\hat{\mathbf{n}}$ is

$$\frac{d\mathcal{W}}{d\Omega} = \int_{-\infty}^{\infty} dt \frac{d\mathcal{P}}{d\Omega} = \frac{Q^2 r_c^2}{128\pi\varepsilon_0 b^3 \beta}\left(3\sin^2\alpha + \sin^2\theta\right)$$

where r_c is the classical radius of the electron, α the angle between $\hat{\mathbf{n}}$ and $\hat{\mathbf{x}}$ and θ the angle between $\hat{\mathbf{n}}$ and $\hat{\mathbf{z}}$. This consists of a dipole radiation pattern symmetric about the direction of motion $\hat{\mathbf{z}}$ and a stronger dipole radiation pattern symmetric about the axis $\hat{\mathbf{x}}$ of closest approach.
(b) Integrate over all solid angles to show that the total energy radiated by the electron is

$$\mathcal{W} = \oint_{4\pi} d\Omega \frac{d\mathcal{W}}{d\Omega} = \frac{Q^2 r_c^2}{12\varepsilon_0 b^3 \beta}$$