EXAMPLE

CONSTRUCT FUNCTIONS OF GOOD SYMMETRY FOR f^2

$$\left.\begin{array}{l} l_1 = 3 \\ l_2 = 3 \end{array}\right\} \quad 0 \leq L \leq 6 \qquad\qquad \left.\begin{array}{l} s_1 = 1/2 \\ s_2 = 1/2 \end{array}\right\} \quad 0 \leq S \leq 1$$

$$\left.\begin{array}{l} m_{l_1} = -3, -2, -1, 0, 1, 2, 3 \\ m_{l_2} = -3, -2, -1, 0, 1, 2, 3 \end{array}\right\} \qquad M_L = \sum_{i=1}^{N} m_{l_i}$$

$$m_s = \pm \frac{1}{2} \qquad\qquad M_S = \sum_{i=1}^{N} m_{s_i}$$

$M_S \backslash M_L$	6	5	4	3	2	1	0
1	–	$\lvert\overset{+}{3}\,\overset{+}{2}\rvert$ 1	$\lvert\overset{+}{3}\,\overset{+}{1}\rvert$ 1	$\lvert\overset{+}{3}\,\overset{+}{0}\rvert$ $\lvert\overset{+}{2}\,\overset{+}{1}\rvert$ 2	$\lvert\overset{+}{3}\,-\overset{+}{1}\rvert$ $\lvert\overset{+}{2}\,\overset{+}{0}\rvert$ 2	$\lvert\overset{+}{1}\,\overset{+}{0}\rvert$ $\lvert\overset{+}{2}\,-\overset{+}{1}\rvert$ $\lvert\overset{+}{3}\,-\overset{+}{2}\rvert$ 3	$\lvert\overset{+}{1}\,-\overset{+}{1}\rvert$ $\lvert\overset{+}{2}\,-\overset{+}{2}\rvert$ $\lvert\overset{+}{3}\,-\overset{+}{3}\rvert$ 3
0	$\lvert\overset{+}{3}\,\bar{3}\rvert$ 1	$\lvert\overset{+}{3}\,\bar{2}\rvert$ $\lvert\bar{3}\,\overset{+}{2}\rvert$ 2	$\lvert\overset{+}{2}\,\bar{2}\rvert$ $\lvert\overset{+}{3}\,\bar{1}\rvert$ $\lvert\bar{3}\,\overset{+}{1}\rvert$ 3	$\lvert\overset{+}{3}\,\bar{0}\rvert$ $\lvert\bar{3}\,\overset{+}{0}\rvert$ $\lvert\overset{+}{2}\,\bar{1}\rvert$ $\lvert\bar{2}\,\overset{+}{1}\rvert$ 4	$\lvert\overset{+}{1}\,\bar{1}\rvert$ $\lvert\overset{+}{3}\,-\bar{1}\rvert$ $\lvert\bar{3}\,-\overset{+}{1}\rvert$ $\lvert\overset{+}{2}\,\bar{0}\rvert$ $\lvert\bar{2}\,\overset{+}{0}\rvert$ 5	$\lvert\overset{+}{1}\,\bar{0}\rvert$ $\lvert\bar{1}\,\overset{+}{0}\rvert$ $\lvert\overset{+}{2}\,\bar{1}\rvert$ $\lvert\bar{2}\,\overset{+}{1}\rvert$ $\lvert\overset{+}{3}\,-\bar{2}\rvert$ $\lvert\bar{3}\,-\overset{+}{2}\rvert$ 6	$\lvert\overset{+}{3}\,-\bar{3}\rvert$ $\lvert\bar{3}\,-\overset{+}{3}\rvert$ $\lvert\overset{+}{2}\,-\bar{2}\rvert$ $\lvert\overset{+}{2}\,\overset{+}{2}\rvert$ $\lvert\overset{+}{1}\,-\bar{1}\rvert$ $\lvert\bar{1}\,-\overset{+}{1}\rvert$ $\lvert\overset{+}{0}\,\bar{0}\rvert$ 7

SPECTROSCOPIC TERM: $\quad {}^{2S+1}L$

$M_S = -S, -S+1, \ldots\ S-1, S \qquad (2S+1)$

$M_L = -L, -L+1, \ldots\ L-1, L \qquad (2L+1)$

$M_S \backslash M_L$	6	5	4	3	2	1	0	6	5	4	3	2	1	0	6	5	4	3	2	1	0
1	–	1	1	2	2	3	3	–	1	1	2	2	3	3	–	0	0	1	1	2	2
0	1	2	3	4	5	6	7	0	1	2	3	4	5	6	0	0	1	2	3	4	5
				^1I							^3H							^1G			

	6	5	4	3	2	1	0	6	5	4	3	2	1	0	6	5	4	3	2	1	0	6	5	4	3	2	1	0
1	–	0	0	1	1	2	2	–	0	0	0	0	1	1	–	0	0	0	0	1	1	0	0	0	0	0	0	0
0	0	0	0	1	2	3	4	0	0	0	0	1	2	3	0	0	0	0	0	1	2	0	0	0	0	0	0	1
				^3F							^1D							^3P							^1S			

SPECTROSCOPIC TERMS FOR f^2:

$$^3P \; ^3F \; ^3H \quad ^1S \; ^1D \; ^1G \; ^1I$$

^1I: $S=0$ $L=6$, $M_L=6$: $|\overset{+}{3}\bar{3}|$
$M_S=0$

$$|^1I\,0\,6\rangle \equiv |\overset{+}{3}\bar{3}|$$

$$|^{2S+1}L\,M_S M_L\rangle$$

FIND APPROPRIATE LINEAR COMBINATION OF DETERMINANTAL STATES TO GIVE

$$|^1I\,0\,5\rangle$$

$$\begin{cases} L_\pm |LM\rangle = \sqrt{L(L+1)-M(M\pm1)}\;|L\,M\pm1\rangle \\ L_\pm = \sum_{i=1}^{N} l_{\pm i} \end{cases}$$

$$L_-|^1I\,0\,6\rangle = \sqrt{6\cdot7-6\cdot5}\;|^1I\,0\,5\rangle = \sqrt{12}\;|^1I\,0\,5\rangle$$

$L=6$
$M_L=6$

$$\| \quad |\overset{+}{3}\bar{3}|$$

$$L_-|\overset{+}{3}\bar{3}| = (l_{1-}+l_{2-})|\overset{+}{3}\bar{3}| = \sqrt{3\cdot4-3\cdot2}\;|\overset{+}{2}\bar{3}| + \sqrt{3\cdot4-3\cdot2}\;|\overset{+}{3}\bar{2}| =$$

$$= \sqrt{6}\left(|\overset{+}{2}\bar{3}| + |\overset{+}{3}\bar{2}|\right)$$

$$\sqrt{12}\,|^1I\,0\,5\rangle = \sqrt{6}\left(|\overset{+}{2}\,\bar{3}| + |\overset{+}{3}\,\bar{2}|\right)$$

$$|^1I\,0\,5\rangle = \frac{1}{\sqrt{2}}\left(|\overset{+}{2}\,\bar{3}| + |\overset{+}{3}\,\bar{2}|\right)$$

BUT $|\overset{+}{2}\,\bar{3}|$ AND $|\overset{+}{3}\,\bar{2}|$ ARE ALSO COMPONENTS OF $|^3H\,0\,5\rangle$

$\langle ^1I\,0\,5|^3H\,0\,5\rangle = 0$ ORTHOGONAL

$$a_1|\overset{+}{2}\,\bar{3}| + a_2|\overset{+}{3}\,\bar{2}|$$

$$\frac{1}{\sqrt{2}}\left(|\overset{+}{2}\,\bar{3}| + |\overset{+}{3}\,\bar{2}|\right)$$

$$\frac{1}{\sqrt{2}}(a_1+a_2)=0 \Rightarrow a_1=-a_2$$

$$\langle ^3H\,0\,5|^3H\,0\,5\rangle = a_1^2+a_2^2=1 \Rightarrow 2a^2=1$$

$$a_1=\frac{1}{\sqrt{2}} \quad a_2=-\frac{1}{\sqrt{2}}$$

$$|^3H\,0\,5\rangle = \frac{1}{\sqrt{2}}\left(|\overset{+}{2}\,\bar{3}| - |\overset{+}{3}\,\bar{2}|\right)$$

$$S_+|^3H\,0\,5\rangle = \sqrt{1\cdot 2-0}\,|^3H\,1\,5\rangle = \sqrt{2}\,|^3H\,1\,5\rangle$$

$$(S_{1+}+S_{2+})\frac{1}{\sqrt{2}}\left(|\overset{+}{2}\,\bar{3}| - |\overset{+}{3}\,\bar{2}|\right) = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{1}{2}\cdot\frac{3}{2}-\left(-\frac{1}{2}\right)\cdot\frac{1}{2}}\,|\overset{+}{2}\,\overset{+}{3}| - \sqrt{\frac{1}{2}\cdot\frac{3}{2}-\left(-\frac{1}{2}\right)\cdot\frac{1}{2}}\,|\overset{+}{3}\,\overset{+}{2}|\right)=$$

$$= \frac{1}{\sqrt{2}}\left(|\overset{+}{2}\,\overset{+}{3}| - |\overset{+}{3}\,\overset{+}{2}|\right) = \frac{2}{\sqrt{2}}|\overset{+}{2}\,\overset{+}{3}| = \sqrt{2}\,|\overset{+}{2}\,\overset{+}{3}|$$

$$|^3H\,1\,5\rangle = |\overset{+}{2}\,\overset{+}{3}|$$