Problems IV

1. **Evaluate the following matrix elements**
   (notation: nl_m)

   $\langle 2p_1 | C^{(1)}_2 | 2p_{-1} \rangle$ $\qquad$ $\langle 3p_1 | C^{(1)}_0 | 4f_1 \rangle$

   $\langle 2p_0 | C^{(2)}_0 | 3p_0 \rangle$ $\qquad$ $\langle 5g_{-4} | C^{(4)}_{-4} | 3d_0 \rangle$

   $\langle 2p_{-1} | C^{(4)}_2 | 2p_{-1} \rangle$ $\qquad$ $\langle 5g_4 | C^{(4)}_2 | 3d_2 \rangle$

2. **Derive the selection rules for the non-vanishing matrix elements**

   $\langle 4f_3 | C^{(k)}_q | 4f_3 \rangle$

   $\langle 1s_0 | C^{(3)}_2 | nl_m \rangle$

3. **Evaluate the following matrix elements; find appropriate values of q in each case**

   $\langle {}^3H_4\ 4 | J^{(1)}_q | {}^3H_4\ 3 \rangle$ *(notation: $^{2S+1}L_J M_J$)*

   $\langle {}^3H\ 15 | S^{(1)}_q | {}^3H\ 15 \rangle$ *(notation: $^{2S+1}L\, M_S M_L$)*

   $\langle {}^3H\ 15 | L^{(1)}_q | {}^3H\ 15 \rangle$ *(notation: $^{2S+1}L\, M_S M_L$)*

4. **Find the relation between the scalar and tensorial products of two tensor operators**

5. **Evaluate the energy of the Coulomb interaction, $E(^{2S+1}L)$, in the case of the following spectroscopic terms of $4f^2$ configuration:**

$$^1S,\ ^3P,\ ^1D,\ ^3F,\ ^1G,\ ^3H,\ ^1I$$