

# FRACTIONAL PARENTAGE COEFFICIENTS

$$|L^N S M_S L M_L\rangle$$

( $\Rightarrow$  DETERMINANTAL FUNCTIONS)



QUANTUM NUMBERS ( $[H, S^2]=0, [H, S_z]=0, [L^2, H]=0, [L_z, H]=0$ )

IN GENERAL: THE FUNCTIONS IDENTIFIED BY THESE NUMBERS ARE THE BASIS FUNCTIONS OF THE IRREDUCIBLE REPRESENTATIONS OF CERTAIN GROUPS (FOR  $R_3$ : GROUP OF ROTATIONS GENERATED BY THE ANGULAR MOMENTUM OPERATORS)

RACAH: TO EVALUATE A MATRIX ELEMENT...

$$\chi S M_S L M_L \equiv \Omega : L^N \quad \text{DAUGHTERS (OFFSPRINGS)}$$

$$\bar{\chi} \bar{S} \bar{M}_S \bar{L} \bar{M}_L \equiv \bar{\Omega} : L^{N-1} \quad \text{PARENTS}$$

$$m_s m_l \equiv \omega : l$$

$$|\Omega\rangle = \sum_{\bar{\Omega}, \omega} (\bar{\Omega}; \omega | \Omega) |\bar{\Omega}\rangle |\omega_N\rangle$$

REFERS TO THE N-TH PARTICLE

$$(L^{N-1} \bar{\Omega} | L^N \Omega) \quad \text{C.F.P.}$$

COEFFICIENTS OF FRACTIONAL PARENTAGE

„THE REST“ COUPLED TOGETHER

## 10. SINGLE-PARTICLE OPERATOR

$$\langle \Omega | F | \Omega' \rangle = N \langle \Omega | f_N | \Omega' \rangle \quad \text{IF } F = \sum_i^N f_i$$

$$\langle \Omega | F | \Omega' \rangle = N \sum_{\substack{\bar{\Omega}, \omega \\ \bar{\Omega}', \omega'}} (\Omega | \bar{\Omega}; \omega) (\bar{\Omega} | \bar{\Omega}') (\omega_N | f_N | \omega'_N) (\bar{\Omega}'; \omega' | \Omega')$$

$\delta(\bar{\Omega}, \bar{\Omega}') \quad ( \text{SAME PARENTS?!} )$

$$\langle \Omega | F | \Omega \rangle = N \sum_{\substack{\bar{\Omega}, \omega, \\ \omega'}} (\Omega | \bar{\Omega}; \omega) (\omega_N | f_N | \omega'_N) (\bar{\Omega}; \omega' | \Omega')$$

$(\Omega | \bar{\Omega})$   
 c.f.p.

$(\bar{\Omega} | \Omega')$   
 c.f.p.

PHASE CONVENTION:

c.f.p.'s ARE REAL  $\Rightarrow (\bar{\Omega}; \omega | \Omega) = (\Omega | \bar{\Omega}; \omega)$

2° TWO-PARTICLE OPERATOR

$\pi : \mathcal{L}^{N-2}$   
 $\pi : \mathcal{L}^2$

$$|\Omega\rangle = \sum_{\pi, \pi'} (\pi; \pi | \Omega) |\pi\rangle |\pi_{NN-1}\rangle$$

TWO PARTICLES UNCOUPLED

$$\langle \Omega | G | \Omega' \rangle = \frac{1}{2} N(N-1) \langle \Omega | g_{NN-1} | \Omega' \rangle =$$

$$\frac{1}{2} N(N-1) \sum_{\substack{\pi \pi' \\ \pi \pi'}} (\Omega | \pi; \pi) (\pi | \pi') \langle \pi_{NN-1} | g_{NN-1} | \pi'_{NN-1} \rangle (\pi'; \pi' | \Omega')$$

1...N-2      N-1 N

$\delta(\pi, \pi')$

$$\langle \Omega | G | \Omega' \rangle = \frac{1}{2} N(N-1) \sum_{\pi, \pi'} \langle \Omega | \pi; \pi \rangle \langle \pi_{NN-1} | g_{NN-1} | \pi'_{NN-1} \rangle (\pi; \pi' | \Omega')$$

c.f.p. :

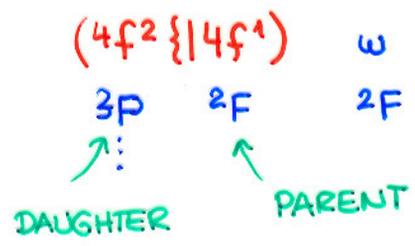
$(\mathcal{L}^{N-1} \times \mathcal{S} \times \mathcal{M}_S \times \mathcal{L} \times \mathcal{M}_L; \ell m_s m_L | \mathcal{L}^N \times \mathcal{S} \times \mathcal{M}_S \times \mathcal{L} \times \mathcal{M}_L)$

↑  
 (COUPLING AND RECOUPLING COEFFICIENTS)

[ONE PARTICLE FRACTIONAL PARENTAGE COEFFICIENTS F2]//Range: from 3P to

4f<sup>2</sup>:

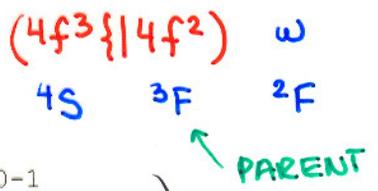
1I			
$\Omega$	$\bar{\Omega}$	$\omega$	
3P[DAUGHTER TERM]	2F	2F	1
3F[DAUGHTER TERM]	2F	2F	1
3H[DAUGHTER TERM]	2F	2F	1
1S[DAUGHTER TERM]	2F	2F	1
1D[DAUGHTER TERM]	2F	2F	1
1G[DAUGHTER TERM]	2F	2F	1
1L[DAUGHTER TERM]	2F	2F	1



[ONE PARTICLE FRACTIONAL PARENTAGE COEFFICIENTS F3]//Range: from 4S to

4f<sup>3</sup>

2L			
$\Omega$	$\bar{\Omega}$	$\omega$	
4S[DAUGHTER TERM]	3F	2F	1
4D[DAUGHTER TERM]	3P	2F	-1
	3F	2F	-1
	3H	2F	-1
4F[DAUGHTER TERM]	3P	2F	1
	3F	2F	-1
	3H	2F	1
4G[DAUGHTER TERM]	3P	2F	1
	3F	2F	-1
	3H	2F	-1
4I[DAUGHTER TERM]	3F	2F	-1
	3H	2F	1



} "POWERS OF PRIMES" !

∴    1    2357    1113...

# LITERATURE:

**EDMONDS, A. R., 1996**, "Angular Momentum in Quantum Mechanics,"  
*Princeton, NJ: Princeton University Press.*

**JUDD, B. R., 1963**, "Operator Techniques In Atomic Spectroscopy,"  
*New York: McGraw-Hill Book Company, Inc.*

**JUDD, B. R., 1967**, "Second Quantization And Atomic Spectroscopy,"  
*Baltimore, MD: The Johns Hopkins Press.*

**JUDD, B. R., 1968**, "Group Theory In Atomic Spectroscopy,"  
*New York, NY: Academic Press.*

**NIELSON, C.W. and KOSTER, G.F., 1963**, "Spectroscopic Coefficients for the  
 $p^n$ ,  $d^n$ , and  $f^n$  Configurations",  
*Cambridge, MA: The M. I. T. Press*

**RACAH, G., 1942**, "Theory Of Complex Spectra. I,"  
*Phys. Rev.*, **61** pp 134--45

**RACAH, G., 1942**, "Theory Of Complex Spectra. II,"  
*Phys. Rev.*, **62** pp 146--70

**RACAH, G., 1943**, "Theory Of Complex Spectra. III,"  
*Phys. Rev.*, **63** pp 367--82

**RACAH, G., 1949**, "Theory Of Complex Spectra. IV,"  
*Phys. Rev.*, **76** pp 1352--65

**RACAH, G., 1951**, "Group Theory And Spectroscopy,"  
"Springer Tracts In Modern Physics, Vol. 37," **1965**, pp 28--84 *New York, NY:*  
*Springer-Verlag*

**ROTENBERG et al, 1959**, "The 3-j and 6-j Symbols,"  
*The Technology Press, Massachusetts Institute of Technology.*

**WYBOURNE, B. G., 1970**, "Symmetry Principles And Atomic Spectroscopy,"  
*New York, NY: Wiley-Interscience -- A Division of John Wiley & Sons.*