FRACTIONAL PARENTAGE COEFFICIENTS

$|l^N S M_S L M_L\rangle$ (≡> DETERMINANTAL FUNCTIONS)

QUANTUM NUMBERS ($[H,S^2]=0$, $[H,S_z]=0$, $[L^2,H]=0$ $[L_z,H]=0$)

IN GENERAL: THE FUNCTIONS IDENTIFIED BY THESE NUMBERS ARE THE BASIS FUNCTIONS OF THE IRREDUCIBLE REPRESENTATIONS OF CERTAIN GROUPS (FOR R_3: GROUP OF ROTATIONS GENERATED BY THE ANGULAR MOMENTUM OPERATORS)

RACAH: TO EVALUATE A MATRIX ELEMENT...

$\gamma S M_S L M_L \equiv \Omega : l^N$ DAUGHTERS (OFFSPRINGS)

$\bar{\gamma} \bar{S} \bar{M}_S \bar{L} \bar{M}_L \equiv \bar{\Omega} : l^{N-1}$ PARENTS

$m_s m_l \equiv \omega : l$

$$|\Omega\rangle = \sum_{\bar{\Omega},\omega} (\bar{\Omega};\omega|\Omega)\, |\bar{\Omega}\rangle |\omega_N\rangle$$

↖ REFERS TO THE N-TH PARTICLE

$(l^{N-1}\bar{\Omega}\}\{l^N\Omega)$ C.f.p.

COEFFICIENTS OF FRACTIONAL PARENTAGE

„THE REST" COUPLED TOGETHER

1°. SINGLE-PARTICLE OPERATOR

$$\langle\Omega|F|\Omega'\rangle = N\langle\Omega|f_N|\Omega'\rangle \quad \text{IF} \quad F=\sum_i^N f_i$$

$$\langle\Omega|F|\Omega'\rangle = N \sum_{\substack{\bar{\Omega}\omega \\ \bar{\Omega}'\omega'}} (\Omega|\bar{\Omega};\omega)(\bar{\Omega}|\bar{\Omega}')(\omega_N|f_N|\omega_N')(\bar{\Omega}';\omega'|\Omega')$$

$1,2,\dots N$ $\quad$ $1,2,\dots N-1$ $\quad$ N

$\delta(\bar{\Omega},\bar{\Omega}')$ (SAME PARENTS?!)

$$\langle \Omega | F | \Omega \rangle = N \sum_{\substack{\bar{\Omega}, \omega, \\ \omega'}} (\Omega | \bar{\Omega}; \omega)(\omega_N | f_N | \omega'_N)(\bar{\Omega}; \omega' | \Omega')$$

$(\Omega\{|\bar{\Omega})$ c.f.p. $\qquad$ $(\bar{\Omega}|\}\Omega')$ c.f.p.

PHASE CONVENTION:

C.f.p.'s ARE REAL $\Rightarrow$ $(\bar{\Omega}; \omega | \Omega) = (\Omega | \bar{\Omega}; \omega)$

2° TWO-PARTICLE OPERATOR

$\Pi : l^{N-2}$

$\pi : l^2$

$$|\Omega) = \sum_{\Pi, \pi} (\Pi; \pi | \Omega) |\Pi\rangle |\pi_{N\,N-1}\rangle$$

TWO PARTICLES UNCOUPLED

$$\langle \Omega | G | \Omega' \rangle = \frac{1}{2} N(N-1) \langle \Omega | g_{NN-1} | \Omega' \rangle =$$

$1, 2, \ldots N$

$$\frac{1}{2} N(N-1) \sum_{\substack{\Pi\Pi' \\ \pi\pi'}} (\Omega | \Pi; \pi)(\pi | \pi') \langle \pi_{NN-1} | g_{NN-1} | \pi'_{NN-1} \rangle (\Pi'; \pi' | \Omega')$$

$1 \ldots N-2$ $\qquad$ $N-1\ N$

$\delta(\Pi, \Pi')$

$$\langle \Omega | G | \Omega' \rangle = \frac{1}{2} N(N-1) \sum_{\Pi, \pi, \pi'} \langle \Omega | \Pi; \pi \rangle \langle \pi_{NN-1} | g_{NN-1} | \pi'_{NN-1} \rangle (\Pi; \pi' | \Omega')$$

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c.f.p. :

$$(l^{N-1}\, \bar{\gamma}\, \bar{S}\, \bar{M}_S\, \bar{L}\, \bar{M}_L ; l m_s m_l | l^N\, \gamma\, S M_S L M_L)$$

↑

(COUPLING AND RECOUPLING COEFFICIENTS)

[ONE PARTICLE FRACTIONAL PARENTAGE COEFFICIENTS F2]//Range: from 3P to 1I

$4f^2$:

Ω Ω̄ ω

3P[DAUGHTER TERM]
2F 2F 1

3F[DAUGHTER TERM]
2F 2F 1

3H[DAUGHTER TERM]
2F 2F 1

1S[DAUGHTER TERM]
2F 2F 1

1D[DAUGHTER TERM]
2F 2F 1

1G[DAUGHTER TERM]
2F 2F 1

1I[DAUGHTER TERM]
2F 2F 1

$(4f^2\{|4f^1)$ ω

^3P ^2F ^2F

^3P: DAUGHTER

^2F: PARENT

[ONE PARTICLE FRACTIONAL PARENTAGE COEFFICIENTS F3]//Range: from 4S to 2L

$4f^3$

Ω Ω̄ ω

$(4f^3\{|4f^2)$ ω

^4S ^3F ^2F

^3F: PARENT

4S[DAUGHTER TERM]
3F 2F 1

4D[DAUGHTER TERM]
3P 2F -1 0 1 0-1
3F 2F -1 1-2
3H 2F -1 1-2 0-1 1

4F[DAUGHTER TERM]
3P 2F 1 -1 0 0-1
3F 2F -1 1-1
3H 2F 1 -1-1 0-1 1

4G[DAUGHTER TERM]
3P 2F 1 -1-1 0-1 1
3F 2F -1 1-2
3H 2F -1 -1-2 1-1 0 1

4I[DAUGHTER TERM]
3F 2F -1 1-2
3H 2F 1 0-2 0 1

⋮ 1 2357 1113...

"POWERS OF PRIMES"!

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