I. (30) Least Squares. Statistician Marge Inovera has collected some data $\left(x_{i}, y_{i}\right)$ which she thinks should follow the relationship, $y=a x+b \exp \left(3 x^{2}\right)$.
A. Help Marge with her analysis by obtaining the least-squares equations for an unweighted fit of data to this equation. Then express these using matrix notation. (Note: It is NOT necessary to solve these equations. Assume, as we have always done, that $x$ is error-free.)
B. Alternatively, Marge could modify the fit relation to permit use of a straight-line fitting routine. If she chooses to fit to the form $Y=a+b X$, how should she define $Y$ and $X$ in terms of $y$ and $x$ ?
C. In the latter analysis, how should the data be weighted?
D. A quantity is measured 29 times, yielding 12.3456 for its mean and $\mathbf{S}=0.7114$ for its sum of squared residuals. What is the approximate uncertainty of measurement $\left(s_{y}\right)$ for this quantity? If you repeated the experiment but did 290 measurements, what (approximately) would you expect for $\mathbf{S}$ and $s_{y}$ ?
II. (16) Conjugated Dye Absorption. One of the conjugated dyes we studied in the laboratory has a molar mass of $454.4 \mathrm{~g} / \mathrm{mol}$.
A. A stock solution is prepared by dissolving $5.3 \pm 0.2 \mathrm{mg}$ of dye in 25.0 mL of methanol. Calculate the concentration of this solution and its uncertainty, and report the results correctly.
B. $\quad 3.00 \mathrm{~mL}$ of this solution is placed in a second $25.0-\mathrm{mL}$ volumetric flask, which is then filled to the mark with methanol. When placed in a $2.00-\mathrm{mm}$ cuvette, this solution yields a peak absorbance of 3.45. Assuming this measurement is valid, what would you expect for the peak absorbance of a 3rd sample, prepared by placing 2.00 mL of the 2 nd solution in a $25.0-\mathrm{mL}$ vol flask and filling to the mark, if this sample is measured in a $1.000-\mathrm{cm}$ cuvette.
C. A conjugated dye like those we studied has 5 C atoms in its chromophoric chain and shows peak absorbance at 535 nm . From the simple theory, where do you expect the peak to occur for the dye having 9 C atoms in its chromophoric chain? (Treat wavelengths as vacuum.)
III. (30) Atomic Term Notation. Consider the ${ }_{5} \mathrm{~B}$ atom.
A. Give the ground configuration the ground term, and the ground state.
B. The first excited configuration of B is obtained by promoting a $2 s$ electron to $2 p$. Give this configuration.
C. Using what we found in class for the ground configuration of ${ }_{6} \mathrm{C}$, work out all the terms of this excited configuration. [Hint: Use the C results for $L_{12}$ and $S_{12}$ and then couple on the 2 s electron.]
D. Compute the total degeneracy of this excited configuration from the standpoint of the available orbitals. Then confirm that you get the same results by considering (1) the $S$ and $L$ degereracies of all terms, and (2) the summed $J$ degeneracies.
E. Give all the eigenvalues for $\hat{S}^{2}, \hat{S}_{z}, \hat{L}^{2}, \hat{L}_{z}, \hat{J}^{2}$, and $\hat{J}_{z}$, operating on the states of a ${ }^{3} \mathrm{D}$ term.
IV. (12) Significance.
A. Consider the following two cases: (1) In 1948 Babcock and Herzberg analyzed an electronic absorption transition in $\mathrm{O}_{2}$ and obtained for the ground-state rotational constant, $B_{0}{ }^{\prime \prime}=$ $1.437770(15) \mathrm{cm}^{-1}$, whereas in 1968 McKnight and Gordy used microwave spectroscopy to obtain $B_{0}{ }^{\prime \prime}=1.437681(1) \mathrm{cm}^{-1}$. (2) In 1976 Tellinghuisen, et al. estimated the dissociation energy of the ground state of XeCl to be $255(10) \mathrm{cm}^{-1}$, whereas later analysis by Sur, et al. yielded $\mathcal{D}_{e}=281(10)$ $\mathrm{cm}^{-1}$.
Which of these two cases constitutes the greater cause for "concern," and why?
B. This question concerns the KaleidaGraph output shown to the right, for an unweighted least-squares fit.
(a) Report each result and its error with the proper number of significant figures.
(b) Are any of the fit parameters statistically insignificant? Explain.

| $y=a^{*} \exp \left(b^{*}(x-20)+c^{*}(x-20 \ldots\right.$ |  |  |
| ---: | ---: | ---: |
|  | Value | Error |
| $a$ | 28.513702 | 0.0002869884 |
| $b$ | 0.0010948899 | $6.41379 \mathrm{e}-06$ |
| c | $-1.909398 \mathrm{e}-06$ | $5.418037 \mathrm{e}-06$ |
| Chisq | $3.6412854 \mathrm{e}-06$ | NA |
| R | 0.99989437 | NA |

V. (12) Spectroscopic Fundamentals. A particular atom has four energy levels, labeled 1-4 in order of increasing energy. A transition between levels 1 and 3 occurs at 246 nm , while one between levels 2 and 4 falls at 410 nm (both measured in vacuum). Level 4 is $11430 \mathrm{~cm}^{-1}$ above level 3. Calculate the wavenumber and wavelength (vacuum) of a transition between levels 1 and 2.

## Fundamental Constants

$$
\begin{aligned}
& N_{\mathrm{A}}=6.0221420 \times 10^{23} / \mathrm{mol} \\
& e=1.602176462 \times 10^{-19} \mathrm{C}
\end{aligned}
$$

$$
c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}
$$

$$
m_{e}=9.1093819 \times 10^{-31} \mathrm{~kg}
$$

$$
\begin{aligned}
h & =6.6260688 \times 10^{-34} \mathrm{~J} \mathrm{~s} \\
m_{n} & =16726216 \times 10^{-27 \mathrm{~kg}}
\end{aligned}
$$

$$
m_{p}=1.6726216 \times 10^{-27} \mathrm{~kg}
$$

