

**Pledge and signature:**

**Note:** If you want your paper returned folded (i.e., score concealed), please print your name on the back.

1. (7) Consider the probability distribution,  $P(x) = c x^2$ , defined over the range  $-1 \leq x \leq 2$ . For this distribution, calculate: (a) the normalization constant, (b) the mean, (c) the variance, and (d) the standard deviation.
 

(a)  $1/3$       (b)  $5/4$       (c)  $11/5 - (5/4)^2 = 51/80$       (d)  $(51/80)^{1/2}$
2. (6) (a) If you generate  $10^6$  random numbers having this distribution, how many are expected to fall within the  $x$  range 1.40–1.50? And what is the standard deviation of this value?  
 (b) If you now generate  $10^6$  such random numbers, what do you get in place of your results in 2a?  
 (c) Compare the *per cent* standard deviations in a and b.
 

(a) 7011 and  $(7011)^{1/2}$       (b) 70111 and  $(70111)^{1/2}$       (c) 1.19% & 0.38%
3. (5) A quantity  $x$  is uncertain by 3.0% and  $y$  is uncertain by 4.0%. Give the % uncertainties for  $z$  in each of the following cases:
 

a. $z = 9/y$ 4 %	d. $z = 5 x/y^2$ $(73)^{1/2}$ %
b. $z = 3 x^4$ 12 %	e. $z = 23 y^2/x$ $(73)^{1/2}$ %
c. $z = 1/\sqrt{8x}$ 1.5 %	
4. (9) **Least Squares and KaleidaGraph.**
  - The declining exponential function with a background is a very commonly occurring functional form in the analysis of kinetics data. Write **exactly** what you should enter in the KG Define Fit box to fit your kinetics data to this relation
 

$a * \exp(-b * x) + c$ ;  $a$  = (nonzero value);  $b$  = (same);  $c$  = (same)
  - Why are bad initial values likely to give you more problems here than in, say, fitting calibration data to a quadratic polynomial?  
 This is a truly **nonlinear** fit, whereas the polynomial fit is algebraically linear (hence guaranteed to converge for any acceptable starting values).
  - In one of your KG exercises, you generated  $10^4$  sums of 12 random numbers. Describe the shape of the resulting histogram, and give the expected mean and standard deviation.

Very nearly Gaussian, mean 6, st. dev. 1. [mean and for **sum** = 12\*(values for **avgs** = 1/2 and 1/12).]

- Suppose instead you generated  $10^4$  sums of 16 random numbers. How would the results change? (Be quantitative.)

Gaussian, mean 8, st. dev. =  $2/3^{1/2}$  [again, both =  $16 * (\text{values for } \text{avgs} = 1/2 \text{ and } 1/(4 \sqrt{12}))$ .]