

September 30, 2009 — Probability Distributions and Related Statistics

Pledge and signature:**Note:** If you want your paper returned folded (i.e., score concealed), please print your name on the back.

1. (8) Consider the probability distribution, $P(x) = cx^2$, defined over the range $0 \leq x \leq 2$. For this distribution, calculate: (a) the normalization constant, (b) the mean, (c) the standard deviation, and (d) the median.

(a) $c = 3/8$ (b) mean = $3/2$ (c) var = $3/20$ $\sigma = (3/20)^{1/2}$

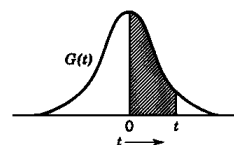
(d) median = $4^{1/3}$

2. (4) If you generate 10^5 random numbers using this distribution, how many would you expect to fall within the range 1.40–1.50? What is the standard deviation of this value?

number = 7888

sd = 89 (Poisson approx)

Table 4-4. Error Function $\frac{1}{2} \operatorname{erf}(t)$ from 0 to t and Ordinate Values
 $G(t) = (1/\sqrt{2\pi}) e^{-t^2/2}$



t	$\frac{1}{2} \operatorname{erf}(t)$ Area	$G(t)$ Ordinate	t	$\frac{1}{2} \operatorname{erf}(t)$ Area	$G(t)$ Ordinate
0	0	0.3989	2.0	0.4773	0.0540
0.1	0.0398	0.3970	2.1	0.4821	0.0440
0.2	0.0793	0.3910	2.2	0.4861	0.0355
0.3	0.1179	0.3814	2.3	0.4893	0.0283
0.4	0.1554	0.3683	2.4	0.4918	0.0224
0.5	0.1915	0.3521	2.5	0.4938	0.0175
0.6	0.2258	0.3332	2.6	0.4953	0.0136
0.7	0.2580	0.3123	2.7	0.4965	0.0104
0.8	0.2881	0.2897	2.8	0.4974	0.0079
0.9	0.3159	0.2661	2.9	0.4981	0.0060
1.0	0.3413	0.2420	3.0	0.4987	0.0044
1.1	0.3643	0.2179	3.1	0.4990	0.0033
1.2	0.3849	0.1942	3.2	0.4993	0.0024
1.3	0.4032	0.1714	3.3	0.4995	0.0017
1.4	0.4192	0.1497	3.4	0.4997	0.0012
1.5	0.4332	0.1295	3.5	0.4998	0.0009
1.6	0.4452	0.1109	3.6	0.4998	0.0006
1.7	0.4554	0.0941	3.8	0.4999	0.0003
1.8	0.4641	0.0790	4.0	0.5000	0.0001
1.9	0.4713	0.0656	4.4	0.5000	0.0000

3. (4) You now generate 10^5 *averages* of 21 such random numbers.

a. Calculate the expected mean and standard deviation for these averages.

$$\text{mean} = 3/2 \text{ (same)}$$

$$\sigma_{\text{mean}} = \sigma/21^{1/2} = 0.084515$$

b. Use the table on the first page to estimate how many of the 10^5 averages would fall between 1.40 and 1.50.

38,140

4. (4) You used the `ran()` command in KaleidaGraph to generate random numbers in the range 0–1. Write **exactly** what you entered in the KG Formula Entry box to obtain

a. an average of two random numbers in column c1.

$$c1 = (\text{ran()} + \text{ran()})/2$$

b. the three-step sequence of commands you used to obtain an average of 12 random numbers in c2.

$$(1) \quad c2 = \text{ran()} + \text{ran()} + \text{ran()}$$

$$(2) \quad c2 = c2 + \text{ran()} + \text{ran()} + \text{ran()} \quad [\text{execute 3 times}]$$

$$(3) \quad c2 = c2/12$$

5. (3) In one of your KG3 exercises, you fitted data to a declining exponential function plus a background, requiring three adjustable parameters. Using **a**, **b**, and **c** for these, write **exactly** what you had to enter in the **Define Fit** box to carry out this nonlinear LS fit.

$$a * \exp(-b * x) + c; \text{ (and give initial values)}$$

6. (5) A quantity x is uncertain by 3.0% and y is uncertain by 4.0%. Give the % uncertainties for z in each of the following cases:

a. $z = 9/y$

4.0%

d. $z = 5 x^2/y$

(52)^{1/2} %

b. $z = 3 x^5$

15.0%

c. $z = 1/\sqrt{8y}$

2.0%

e. $z = 23 y/x^2$

(52)^{1/2} %