Chemistry 230 — Fall 1999
Problem Set 1 Solutions

1. $\left(\partial R/\partial R_2\right)_{R_1,R_3} = (R/R_2)^2$. $R = 28.6\,\Omega$; $\left(\frac{\partial R}{\partial R_2}\right) = 0.0204$

2. (a) $(\partial w/\partial z) = (xy)^z \ln(xy)$

   (b) (i) $(\partial w/\partial x) = e^x \cos y$; $(\partial w/\partial y) = -e^x \sin y$

   (ii) $(\partial w/\partial x) = \frac{x}{x^2+y^2}$; $(\partial w/\partial y) = \frac{y}{x^2+y^2}$

3. (a) $\frac{\partial f}{\partial x} = \frac{2f}{x}$

   $\frac{\partial f}{\partial y} = 2f$

   $\frac{\partial f}{\partial z} = 3f$

   $\frac{\partial f}{\partial w} = -4x^2 e^{2y} e^{3z} \sin 4w$

   (b) $f_r = \frac{(z^2-r^2)(2-\cos 2\theta)}{(r^2+z^2)^2}$

   $f_\theta = \frac{2r \sin\theta}{r^2+z^2}$

   $f_z = -\frac{2rz(2-\cos 2\theta)}{(r^2+z^2)^2}$

4. $z = \rho\cos\theta$

   $x = \rho\sin\theta\cos\alpha$

   $y = \rho\sin\theta\sin\alpha$

   $\rho = (x^2+y^2+z^2)^{1/2}$

   $\theta = \cos^{-1}\left[\frac{z}{(x^2+y^2+z^2)^{1/2}}\right]$

   $\alpha = \tan^{-1}(y/x)$

   (a) $\frac{\partial \rho}{\partial x} = \frac{x}{(x^2+y^2+z^2)^{1/2}}$

   (b) $\frac{\partial \theta}{\partial y} = \frac{yz}{(x^2+y^2+z^2)(x^2+y^2)^{1/2}}$

   (c) $\frac{\partial \alpha}{\partial x} = \frac{-y}{x^2+y^2}$

5. $\frac{\partial \vec{R}}{\partial x} = \vec{i} + f_x \vec{k}$ (lies in xz plane)

   $\frac{\partial \vec{R}}{\partial y} = \vec{j} + f_y \vec{k}$ (lies in yz plane)

6. $\frac{dw}{dt} = 2t(t^2+1)^2\left[(4t^2+1)\cos 4t - 2t(t^2+1)\sin 4t\right]$

9. Note: In part (a) we start with $x = x(r,\theta)$ & $y = y(r,\theta)$. Thus we can directly obtain $\frac{\partial x}{\partial r}$, $\frac{\partial x}{\partial \theta}$, etc by directly differentiating, because the $x = x(r,\theta)$ & $y = y(r,\theta)$ contain the full dependence on the new variables, r & θ. The corresponding expressions for dr & $d\theta$ in (b), to be complete, require that the right hand side be in terms of x & y.

10. (a) minimum at $(-3,3,-5)$.

    (b) $(-2,1,3)$ is a saddle point.

12. (a) not exact

    (b) exact. $f = x^2+y^2+xy+C$

    (c) exact. $f = y + e^x(y-x+1) + C$