Chemistry 230
Problem Set # 1 — Fall, 1999

1. Three resistors connected in parallel produce a resistance R given by $R^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$. Find $\partial R/\partial R_2$. Evaluate R and this partial if $R_1 = 100$ ohm, $R_2 = 200$ ohm, and $R_3 = 50$ ohm.

2. (a) Find $\partial w/\partial z$ if $w = (xy)^z$.

   (b) Obtain $\partial w/\partial x$ and $\partial w/\partial y$ for: (i) $w = e^x \cos y$; (ii) $w = \ln (x^2 + y^2)^{1/2}$.

3. Find the partial derivatives of f with respect to each independent variable, for

   (a) $f(x,y,z,w) = x^2\, e^{2y+3z} \cos 4w$; (b) $f(r,\theta,z) = \dfrac{r\,(2 - \cos 2\theta)}{r^2 + z^2}$.

4. Express the spherical coordinates ρ, θ, φ in terms of the Cartesian coordinates x, y, and z; and then determine (a) $\partial\rho/\partial x$, (b) $\partial\theta/\partial y$, and (c) $\partial\varphi/\partial x$.

5. Let the vector $\mathbf{R}$ be $\mathbf{R} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}f(x,y)$, where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors pointed along the x, y, and z axes, respectively. What can you say about the directions of the vectors, (a) $\partial\mathbf{R}/\partial x$, and (b) $\partial\mathbf{R}/\partial y$?

6. Suppose $w = e^{2x+3y} \cos 4z$, and x, y, and z are all functions of t, given by $x = \ln t$, $y = \ln (t^2+1)$, $z = t$. Find dw/dt by

   (a) expressing w explicitly as a function of t and differentiating.

   (b) using the chain rule (with partial differentiation).

7. If we substitute polar coordinates $x = r \cos\theta$ and $y = r \sin\theta$ in a function $w = f(x,y)$, show that (a) $\partial w/\partial r = f_x \cos\theta + f_y \sin\theta$; and (b) $r^{-1}\, \partial w/\partial\theta = -f_x \sin\theta + f_y \cos\theta$ (f_x and f_y are the partials with respect to x and y, respectively).

8. Consider the function $w = x^2 + y^2 + z^2$. Express the total differential dw in terms of the independent variables x, y, and z.

   Now suppose that $x = r \cos s$, $y = r \sin s$, and $z = r$. Express the total differentials dx, dy, and dz in terms of the independent variables r and s, and substitute in your previous result for dw to show that $dw = 4r\, dr$.

   Alternatively, substitute for x, y, and z in w and obtain the exact differential dw in terms of the independent variables r and s. You should obtain the same result.

9. (a) Given $x = r \cos\theta$, $y = r \sin\theta$, express dx and dy in term of dr and $d\theta$. (b) Solve the equations of part (a) for dr and $d\theta$ in terms of dx and dy. (c) In the answer to part (b) suppose A and B are the coefficients of dx and dy in the expression for dr, *i.e.*, $dr = A\, dx + B\, dy$. Verify by direct computation that $A = \partial r/\partial x$ and $B = \partial r/\partial y$, where $r^2 = x^2 + y^2$.

10. In a multivariate function, maxima and minima (if they exist) are found by setting the partial derivative with respect to each independent variable equal to zero and solving the resulting set of equations. Find any maxima or minima for the following functions: (a) $z = x^2 + xy + y^2 + 3x - 3y + 4$; (b) $z = x^2 + xy + 3x + 2y + 5$.

11. (a) If $w = \cos (x+y) + \sin (x-y)$, show that $\partial^2 w/\partial x^2 = \partial^2 w/\partial y^2$.

    (b) If c is a constant and $w = f(x + ct) + g(x - ct)$, where $f(u)$ and $g(v)$ are twice-differentiable functions of $u = x + ct$ and $v = x - ct$, respectively, show that

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2} = c^2\,(f''(u) + g''(v)).$$

    (c) Verify that $w_{xy} = w_{yx}$ for (i) $w = \ln (2x + 3y)$; (ii) $w = xy^2 + x^2y^3 + x^3y^4$.

12. Apply the Euler reciprocity relation to determine if the following are exact differentials: (a) $e^y\, dx + x(e^y+1)\, dy$; (b) $(2x+y)\, dx + (x+2y)\, dy$; (c) $(1+e^x)\, dy + e^x(y-x)\, dx$. If the expression is an exact differential, df, find f.